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THE SYNTAX OF THE WORLD

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Strawson and Davidson have tried to shake our faith in the correspondence theory of truth. Professor Sir Peter Strawson once wrote, "Truth is not a property of symbols; for it is not a property".[1] But whether truth is a property is a red herring, for there is no settled general lore of properties that answers the question.[2] On the other hand, where L is the language of first order elementary number theory, there is a set of those sentences of L true in its standard model.[3] This set is the extension of the predicate "is a true sentence of L".[4]

There is also a set of mothers. Its members are the women who bear the parenthood relation to some child. In

[1] P.F. Strawson, "Truth", in Margaret MacDonald (ed.), *Philosophy and Analysis*, Blackwell, 1954, p. 262.

[2] The obvious first shot is comprehension for properties, which says that every predicate expresses a property. But this misfires for non-self-possession.

[3] Ordinary set theory yields the set of natural numbers. Applied set theory yields a one-one correspondence between natural numbers and truths of L. So applied, set theory yields the set of truths of L by replacement. It would not be easy to avoid this result naturally.

[4] Moreover, this set is of degree $O^{(\omega)}$ of unsolvability, and so infinitely more unsolvable that the set of theorems of first order elementary number theory. It would be at least awkward to try to state this enlightening fact were we denied the set of truths of L.

general different children have different mothers. The correspondence theory of truth says that truths bear a correspondence relation to facts, where in general different facts correspond to different truths. But on Tarski's conception, a sentence of L is true if and only if every (or, equivalently, some) sequence of natural numbers satisfies it.[5] Davidson observes that if satisfaction were correspondence and facts were the sets of sequences satisfying a truth, then exactly one fact would correspond to all truths.[6] This upshot flouts the opinion that in general different facts correspond to different truths, as in general different children have different mothers. But all and only the mothers stand in a relation to a unique species, humanity, for they bear its members. That banality should not shake one's faith in motherhood. Nor, we shall argue, need the fact that all and only the truths of L are satisfied by a unique set, the set of all sequences of natural numbers, shake one's faith in correspondence to fact.

To defend the faith, we must rescue facts. We can do so by using set theory to read quantificational syntax into the very world itself. This will eventually yields a sort of propositions, among which facts will be distinguished through a version of satisfaction by all sequences. If we then rescue expression of a proposition by a sentence, correspondence to fact turns out to be expression of a proposition that is a fact. All the set theory involved is pretty straight forward, but the construction gives one another slant on the role of sequences in truth.

To begin our rescue of facts, let D be any non-empty set. Call D the domain (and pick an arbitrary member d^*

[5] Alfred Tarski, "The Concept of Truth in Formalized Languages", in *Logic, Semantics, Metamathematics*, trans. J.H. Woodger, Clarendon Press, 1956, pp. 152–278.

[6] Donald Davidson, "True to the Facts", reprinted in *Inquiries into Truth and Interpretation*, Clarendon Press, 1984, pp. 37–54.

of D). As usual, ω is the set of all natural numbers. Let X be the set of all sequences of members of D, that is, the set of all functions from ω into D. (Let x^* be the member of X whose value is always d^*.) In Tarski's setting, one often thinks of a sequence as a simultaneous evaluation of all the variables of the language for which one is defining truth, the n^{th} item in the sequence being the value of the n^{th} variable. But in our setting, there is no language in view; in that way, our propositions and facts are language-independent, and our sequences are just rows of members of the domain.

Let $\Re$ be a non-empty set of relations on D. We want $\Re$ to be non-empty so we will get some propositions and facts. $\Re$ can be the set of all relations on D. In that case, when D is infinite, there will be more relations in $\Re$ than predicates in a countable language; and there will be more propositions and facts than truths in a countable language. For each R in $\Re$ there is a unique positive integer $p(R)$, called the polyadicity of R, such that R is a set of ordered $p(R)$-tuples of members of D.

In one scheme, the sentence "Socrates is bald" expresses the proposition that Socrates is bald, and the sentence is true (and the proposition, a fact) if and only if Socrates is bald. One extensional reconstruction of this scheme takes this proposition as the ordered pair whose first member is the set of bald things and whose second member is Socrates, and then counts this proposition as a fact just in case its second member is an element of its first. To allow for polyadicity, suppose F is a k-adic predicate and $n_1, \ldots, n_k$ are names. Suppose the denotations of $n_1, \ldots, n_k$ in a non-empty domain D are $d_1, \ldots, d_k$, and that the extension of F in D is a set S of ordered k-tuples of members of D. Then the atomic sentence '$F_{n_1}, \ldots{}_{n_k}$' expresses the proposition $\langle S, d_1, \ldots, d_k\rangle$, and this is a fact if and only if $\langle d_1, \ldots, d_k\rangle \in S$.

This gives us propositions expressed by atomic sentences of a language as standardly interpreted model theoretically. But what about the rest of the senteces of the language? Suppose the non-atomic sentences are built up using negation, disjunction and universal quantification. Then what are the propositions expressed by

$$Fab \ \vee \ Gbc,$$
$$(\forall x) \ Fax?$$

We might take these sentences as applying the complex predicates

$$Fxy \ \vee \ Fyz$$
$$(\forall x) \ Fyx$$

to, respectively, the triple of the denotations of 'a', 'b', 'c', and the denotation of 'a'. But how are the extensions of these complex predicates related to that of the simple predicate F?

One approach to these questions starts from the set X of all sequences of members of the domain D, that is, all functions from ω into D. The original model had a non-empty set $\Re$ of relations on D, one relation of the appropriate polyadicity for each primitive predicate of the language in view. Let R be one of these relations, and suppose R was motherhood. Quantificational syntax exploits identification and permutation of variables. For example, even if the extension of F is a binary relation, the extension of

$$Fxy \vee Fyz$$

should be a ternary relation, and the triples in it should stand in a certain way to the pairs in the extension of F. Again we need to know where the quantifier in

$$(\forall x)Fxy$$

acts on the extension of F to say how the extension of this complex predicate stands to that of F. Sequences are a way of projecting the syntactical utility of variables of quantification out into the world. Recall that R was motherhood. Let $R(X_5, X_{32})$ be the set of all sequences x in X such that the 6^{th} term in x is the mother of the 33^{rd} term in x. (The annoying plus-one comes from starting our sequences at zero.) At first, $R(X_5, X_{32})$ looks a bit like R with a huge amount of irrelevance hanging off it. But suppose the extensions of 'Fy_1y_2' and 'Fy_2y_3' are $R(X_1, X_2)$ and $R(X_2, X_3)$. Let '$\vee$' name the union function. Then the extension of

$$Fy_1y_2 \vee Fy_2y_3$$

can be taken to be

$$R(X_1, X_2) \ u \ R(X_2, X_3),$$

that is, the set of all sequences whose second term is mother of its third or whose third is mother of its fourth, which is good enough for what we need. Negation names complementation (with respect to X), and while universal quantification is a bit more complex, it is not too bad. And we want to eliminate any mention of a language in the background.

The variables of ordinary quantificational syntax play several parts, each of which we will cast among the sequences. So we have to pay some attention to sequences. For any R in $\Re$, let $R(X_1, \ldots, X_{p(R)})$ be the set of all sequences x in X such that $\langle x(1), \ldots, x(p(R))\rangle$ is a member of R. We call $R(X_1, \ldots, X_{p(R)})$ a sequence-relation, and in a slightly extended sense we say that its polyadicity is also $p(R)$. One might think of $R(X_1, \ldots, X_{p(R)})$ as like writing an n-adic predicate F as $F(x_1, \ldots, x_n)$ with explicit use of the first n variables.

We will want to be able to vary the things here and there in a sequence. To that end, let $i_1, \ldots, i_n$ be natural numbers and let $d_1, \ldots, d_n$ be members of D. Then for any sequence x in X, $x\,^{d_1 \ldots d_n}_{i_1 \ldots i_n}$ is the sequence y in X such that

$$y(j) = \begin{cases} x(j) & \text{if} \quad j \notin \{i_1, \ldots, i_n\} \\ d_k & \text{if} \quad j = i_k \end{cases}$$

We identify, permute and rewrite the variables of ordinary quantificational syntax. To see how to reflect such activities in the world, suppose $R(X_{i_1}, \ldots, X_{i_p})$ is the set of all sequences x in X such that $\langle x(i_1), \ldots, x(i_p)\rangle$ is in the p-adic relation R in $\mathfrak{R}$. Let $\langle k_1, \ldots, k_p\rangle$ be any ordered p-tuple of natural numbers. Then $R(X_{k_1}, \ldots, X_{k_p})$ is the set of all sequences x in X such that $x\,^{x(k_1) \ldots x(k_p)}_{i_1 \ldots i_p}$ is in $R(X_{i_1}, \ldots, X_{i_p})$. We call it the $k_1, \ldots, k_p$ trade for $i_1, \ldots, i_p$ of R, and we say that its polyadicity is the number of members of $\{k_1, \ldots, k_p\}$, which can be less than p. Suppose, for example, that R is binary. Then $R(X_1, X_1)$ is an ersatz for the set of all members d of D that bear R to themselves, and $R(X_2, X_1)$ is in effect the converse of $R(X_1, X_2)$. In this way, identifications and permutations of variables are reflected in the world. One reason for rewriting free variables is so that while the predicates $F(x_1, x_2)$ and $G(x_2, x_3)$ are binary, their conjunction is ternary; trading also reflects such rewriting.[7]

We use these devices to give an inductive definition of the quantificational relations. To start off, any sequence relation is a quantificational relation. We have four ways of going on. If $Q(X_{i_1}, \ldots, X_{i_p})$ is a quantificational relation and $\langle k_1, \ldots, k_p\rangle$ is an ordered p-tuple of natural numbers, then the $k_1, \ldots, k_p$ trade for $i_1, \ldots, i_p$ of Q is

[7] Our devices descend via Hilary Putnam from Paul Bernays' proof of his class theorem.

also a quantificational relation. Its polyadicity is the number of members of $\{k_1, \ldots, k_p\}$. Next, if $Q(X_{i_1}, \ldots, X_{i_p})$ and $S(X_{j_1}, \ldots, X_{j_m})$ are quantificational relations, then so is their union; its polyadicity is the number of numbers among $i_1, \ldots, i_p, j_1, \ldots, j_m$. Third, if $Q(X_{i_1}, \ldots, X_{i_p})$ is a quantificational relation, then so is its complement with respect to X; its polyadicity is that of Q. Finally, if $Q(X_{i_1}, \ldots, X_{i_p})$ is a quantificational relation of positive polyadicity and i is one of $i_1, \ldots, i_p$, then the set of all sequences x in X such that for all d in D, x_i^d is in Q is also a quantificational relation. It is called the universal quantification of Q in i^{th} place, is written $(\forall X_i)\ Q(X_{i_1}, \ldots, X_{i_p})$, and its polyadicity is the predecessor of that of Q. This completes the definition of the quantificational relations.

Let A be a subset of D. Call each member of A an individual. (Construe n-ary functions on D as $n + 1$-ary single valued relations on D.) We are now ready to define propositions. If $Q(X_{i_1}, \ldots, X_{i_p})$ is a quantificational relation of polyadicity p and $t_1, \ldots, t_p$ are individuals, then

$$\langle Q(X_{i_1}, \ldots, X_{i_p}), t_1, \ldots, t_p \rangle$$

is a proposition, the proposition that $Q(t_1, \ldots, t_p)$. The proposition that $Q(t_1, \ldots, t_p)$ is a fact, namely the fact that $Q(t_1, \ldots, t_p)$, if and only if for all sequences x in X, $x_{i_1 \ldots i_p}^{t_1 \ldots t_p}$ is in $Q(X_{i_1}, \ldots, X_{i_p})$.

Given a domain D, pure set theory yields from the relations on D and the members of D as individuals, the totality of propositions and facts on D. This is a theorem of pure set theory. So our propositions and facts are just as extensional as any other sets. But our present construction may be a bit too extensional. Suppose that $R_1 = D = R_2$. Then the fact that $(\forall X_1)R_1(X_1)$ is the fact that $(\forall X_1)R_2(X_1)$, since both are X. Such collapse happens only for facts that are propositions of the form $\langle Q \rangle$, where Q is a quantificational relation of polyadicity zero. Still, on our present construction, the fact that all men are mortal is the fact that all women are mortal, which might seem excessively extensional since the set of men is different from the set of women.

But, at the cost of some abstraction, we can reduce this excess. To illustrate, let f be the function whose value for any natural number is its square, and let g be the function whose value for any natural number is its double. Then $f(2)$ is identical with $g(2)$, while $\langle f, 2\rangle$ is different from $\langle g, 2\rangle$. On this model, let us prise the functions of their arguments among the quantificational relations.

First, let us isolate the functions. Let u be the function whose value for any subsets A and B of X is their union. Let n be the function whose value for any subset A of X is $X - A$. For each i in ω, let q_i be the function whose value for any quantificational relation $Q(X_{i_1}, \ldots, X_{i_p})$ of positive polyadicity where i is among $i_1, \ldots, i_p$ is $(\forall X_i)$ $Q(X_{i_1}, \ldots, X_{i_p})$ (and whose value otherwise is arbitrary, say $\{x^*\}$). For any $i_1, \ldots, i_p, k_1, \ldots, k_p$, let $s\,{}^{i_1 \ldots i_p}_{k_1 \ldots k_p}$ be the function whose value for any quantificational relation $Q(X_{i_1}, \ldots, X_{i_p})$ is the $k_1, \ldots, k_p$ trade for $i_1, \ldots, i_p$ of Q (and whose value otherwise is arbitrary, say $\{x^*\}$ again). Call these functions the operations.

We will call the analogues of $\langle f, 2\rangle$ relational forms, and those of $f(2)$, their values. These we will define inductively, and at the same time we will show that the value of a relational form is always a quantificational relation. To start off, any sequence relation is a relational form, and is its own value, which is thus a quantificational relation. To go on, suppose that r_1, and r_2 are relational forms, that $Q_1(X_{i_1}, \ldots, X_{i_p})$ and $Q_2(X_{j_1}, \ldots, X_{j_m})$ are their values, and that these values are quantificational relations.

Then $\langle s^{i_1 \ldots i_p}_{k_1 \ldots k_p}, r_1\rangle$, $\langle u, r_1, r_2\rangle$, $\langle n, r_1\rangle$ and, if Q_1 is of positive polyadicity and i is among $i_1, \ldots, i_p$, $\langle qi, r_1\rangle$ are relational forms. Their values are, respectively, $Q_1(X_{k_1}, \ldots, X_{k_p})$, the union of Q_1 and Q_2, the complement of Q_1 with respect to X, and $(\forall X_i)Q_1(\forall X_{i_1}, \ldots, X_{i_p})$, which are all quantificational relations.

This time around we define propositions in terms of relational forms. So, let r be a relational form, let p be the polyadicity of its value $Q(X_{i_1}, \ldots, X_{i_p})$, and let $a_1, \ldots, a_p$ be individuals. Then, and only then, is $\langle r, a_1, \ldots, a_p\rangle$ a proposition, the proposition that $Q(a_1, \ldots, a_p)$. This proposition is a fact, the fact that $Q(a_1, \ldots, a_p)$, if and only if $\langle Q(X_{i_1}, \ldots, X_{i_p}), a_1, \ldots, a_p\rangle$ is a fact in the old sense, that is, for any x in X, $x\,{}^{a_1 \ldots a_p}_{i_1 \ldots i_p}$ is in $Q(X_{i_1}, \ldots, X_{i_p})$. Now the fact that all men are mortal is different from the fact that all women are mortal, and for the desired reason, namely, because the set of men is different from the set of women. Aping part of 1.1 in Wittgenstein's *Tractatus*, we might say that the world is the totality of facts.

Our construction of propositions and facts makes no mention of language. Still, our propositions are tailored for expression by sentences. Let $\mathscr{L}$ be a language built up in the usual way from predicates and names by negation, disjunction and universal quantification.

To interpret $\mathscr{L}$ we need a non-empty domain D, a non-empty set $\mathfrak{R}$ of relations on D, a subset A of D, and a function M that assigns predicates in $\mathscr{L}$ to relation in $\mathfrak{R}$ (matching polyadicities) and names in $\mathscr{L}$ to elements of A. Using M, we can read off a unique proposition from each sentence of $\mathscr{L}$. Call this the proposition expressed (under M) by the sentence. Say that the sentence corresponds (under M) to fact if and only if the proposition expressed (under M) by the sentence is a fact. Then the sentence is a true sentence of $\mathscr{L}$ if and only if it corresponds (under M)

to fact.[8] We might also say that each true sentence of $\mathscr{L}$ corresponds to the fact it expresses.

To illustrate, let $\mathscr{L}$ be the language L for elementary number theory, let ω be the domain, and let M assign the identity relation I on ω to the predicate "$x_1 = x_2$" of L. X is now the set of all sequences of natural numbers. Let r be

$$\langle q_1, \langle s_{1\,1}^{1\,2}, I(X_1, X_2)\rangle\rangle$$

Then $\langle r\rangle$ is the proposition expressed by the sentence "$(\forall x_1)(x_1 = x_1)$" under M. The value of r is $(\forall X_1)\, I\,(X_1, X_1)$, so under M, "$(\forall x_1)(x_1 = x_1)$" expresses the proposition that $(\forall X_1)\, I\,(X_1, X_1)$. Then "$(\forall x_1)(x_1 = x_1)$" is a true sentence of L

> if and only if "$(\forall x_1)(x_1 = x_1)$" corresponds to fact
> if and only if the proposition expressed by "$(\forall x_1)(x_1 = x_1)$" is a fact
> if and only if the proposition that $(\forall X_1)\ I(X_1, X_1)$ is a fact
> if and only if for all x in X and all d in ω, x_1^d is in $I(X_1, X_1)$
> if and only if for all d in ω, $d = d$.

This example shows how M and set theory yield a T-sentence for each sentence of L, as Tarski required of adequate definitions of truth.

Some wish to throw away propositions, expression, correspondence, facts and all of truth but T-sentences.[9] But given that names have denotations, variables have a range and predicates have extensions, it is inevitable that sen-

[8] Tarski forestalls the liar paradox by forbidding the predicate "is a true sentence of $\mathscr{L}$" from being a predicate in $\mathscr{L}$.

[9] Paul Horwich, *Truth*, Blackwell, 1990.

tences express propositions, some of which are facts.[10] Given sets, our propositions and facts are inevitable, and given languages too, so are expressions of propositions and correspondence to fact. Sets and languages are here to stay. An nimbus of fact will always shroud those hunched over T-sentences.[11]

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[10] Pretend that "snow" names the stuff s snow, and take the set W of white things as the extension of the predicate "is white". This being so, it remains so for those expressions used in the hackneyed T-sentence

"Snow is white" is true if and only if snow is white,

which thus says that the sentence "Snow is white" bears relations to s, W and to the proposition that $W(s)$, which is a fact.

[11] I am grateful to Dorothy Grover and Peter Hylton for comments on an earlier draft of this paper.

RESUMEN

Davidson desalienta el compromiso ontológico con hechos distintos que correspondan a oraciones verdaderas distintas. Pero, una vez que se han dado extensiones a los nombres y predicados básicos de un lenguaje construido con funciones veritativas y cuantificadores, es inevitable un desarrollo conjuntístico que usando secuencias conduce a hechos distintos que resultan extensiones de oraciones verdaderas distintas. Como dijo Huxley, los hechos no dejan de existir porque los ignoremos.

[*Traducción: Raúl Orayen*]