FREGE'S PROBLEMS WITH 'THE CONCEPT *HORSE*'*

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The Fregean theories of language attempt to answer the following questions:

(1) How do the meaningful parts of a sentence contribute to the sentence's truth value?

and

(2) How do the meaningful parts of a sentence contribute to the thought expressed by the sentence?

Before obtaining answers to (1) or (2), though, Frege must answer two other questions:

(3) What are the meaningful parts of sentences?

and

(4) Which combinations of the meaningful parts of sentences are sentences?

The answers Frege gives to (3) and (4) comprise what I will call the Fregean theory of syntax; his answers to (1) and (2) form the Fregean theory of meaning.

In § II sketch the Fregean theories of syntax and meaning and their answers to (1)-(4). §II urges that in order to main-

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tain the consistency of these theories Frege is *forced* to draw his very sharp distinction between objects and functions, and between their names: to give the distinction up, or to appreciably weaken it, forces —on pain of contradiction— jettisoning the theories. §III sketches the resulting apparent inability of Frege to specify the reference (or sense) of a function-name and rejects some proposals as to how to circumvent the difficulty. Yet I believe all the material for solving the problem without giving up any distinctively Fregean thesis is at hand, and §IV presents such a solution.

I

The Fregean theory of syntax discerns two kinds of meaningful sentence-parts: function-names —which have functions for their references and functions for their senses— and proper names —which have objects for their references and objects for their senses. Functions —and function names— are assigned levels. A function which takes as arguments the references or senses of proper names is first-level; a function-name which takes in its argument places proper names is first-level. Predicates and sentential connectives are such function-names, and their references and senses such functions. A function which takes as arguments first-level functions is second-level; a function-name which takes in its argument places first-level function-names is second-level. Quantifiers are second-level function-names, and their references and senses are second-level functions. And the hierarchy continues in this manner indefinitely. There are also function-names (and functions) of unequal-level; these are function-names of more than one argument-place, whose argument-places are filled with names or function-names of diferent levels. Thus each proper name and equal-level function-name has a *type*: names of objets (proper names) are of type 1, names of first-level functions of one argument are of type 2, names of first-level functions of two arguments are of type 3, and so on. And associated with each n-place function-name is what we might call an *argument-type* $<t_1, \ldots, t_n>$ which indicates the type of expression appropiate to each argument-place. Fregean functions are to have values for all arguments of the appropriate type. First-level functions have values for every object as argument; second-level functions have values for every first-level function as argument; and so on. Unary first-level functions whose values are always truth values are called "concepts" by Frege.[1]

Functions take arguments and yield values. Function-names, similarly, have argument-places which are filled with names of arguments; thus function-names contain gaps marking their argument-places. These gaps —for a first-level funtion-name— Frege fills with the Greek consonants 'ξ' and 'ζ'; these consonants serve two purposes: they "hold argument places open" and they indicate the patterns for completing the function-name. Thus

$$\xi + \xi$$

stands for the unary function which for any argument gives as value the result of adding the argument to itself;

$$\xi + \zeta$$

on the other hand, stands for the binary function which for any pair of arguments yields as value the result of adding those arguments.[2] The gaps in second-level function-names are filled by the Greek consonants 'φ' and 'ψ'. Thus the gap holders of a function-name indicate the type of the expressions appropiate to each argument-place. The universal quantifier,

$$\mathrm{Vx}\, \varphi\, (x),$$

[1] The theory of syntax is sketched in the early portion of *The Basic Laws of Arithmetic.* Edited and translated with an introduction by Montgomery Furth. Berkeley: University of California Press, 1964 (hereafter cited as '*BL*;). Cf. esp. §§1-8, 12, 13, 21-24. Frege does not talk of argument types; this is, though, a natural extension of his use of 'type', *BL*, §23.

[2] Cf. *BL*, §1.

is a second-level function-name. The result of completing it with a one-place first-level function-name —'$\Phi(\xi)$'— namely,

$$\text{V}x\,\Phi(x),$$

is true just if the function that '$\Phi(\xi)$' stands for has the True for its value no matter what object is taken as argumen.[3] Often —as in this case— when a first-level function-name is fitted into an argument-place of a higher-level function-name, the gaps of the first-level function-name are filled with bound variables which the second-level function-name carries along with it.

Frege's answer to (4), then, is this:

> If $\Sigma\, \beta_1, \ldots, \beta_n[\varphi(\beta_1, \ldots, \beta_m), \ldots, \psi(\beta_p, \ldots, \beta_n)]$ is an n-place function-name of argument type $<t_1, \ldots, t_n>$ and $\Phi(\xi_1, \ldots, \xi_m), \ldots, \psi\,(\xi_p, \ldots, \xi_n)$ are proper names or function-names of (respectively) type $t_1, \ldots, t_n$, then $\Sigma\, \beta_1, \ldots, \beta_n[\Phi(\beta_1, \ldots, \beta_m), \ldots, \psi(\beta_p, \ldots, \beta_n)]$ is a proper name, where we understand that sentences, being names of objects (viz. truth values), are proper names, are of type 1.

It is now easy to see how the theory of meaning will proceed. Each proper name, and thus each sentence, will consist of a main function-name whose argument-places are filled with the appropiate kinds of expressions. The reference of the proper name —in the case of a sentence, its truth value— will be the value of the function which is the reference of the main function-name when it takes as arguments those functions and objects which are the references of those expressions which occur in the argument-places of the proper name's main function-name. That is (for simplicity suppressing some gaps and variables):

[3] Cf. *BL*, §§8, 21-23.

If a proper name results from putting the proper names or function-names $n_1, \ldots, n_k$ (respectively) in the argument-places of a k-place function-name Φ, then $\Phi(n_1, \ldots n_k)$ stands for the object which is the value of the function for which Φ stands when the objects or functions for which $n_1, \ldots, n_k$ stands (respectively) are taken as arguments.[4]

And, in an analagous manner, the senses of the parts of a proper name will combine to yield the sense of the whole. In this way the Fregean theory of meaning seeks to answer (1) and (2).

From these principles two principles of substitution immediately follow, one for reference and one for sense:

If in a proper name a constituent meaningful expression is replaced by another having the same reference (or sense), then the reference (or sense) of the entire proper name is not changed.[5]

II

In order for the Fregean theories of syntax and meaning to be consistent it cannot be that names of different types stand for the same thing. For it they could, then there would be no reason why substitution of one for the other in a proper name would not preserve reference, and thus meaningfulness. But this contradicts the theory of syntax. If, for instance, functions could be referred to by gapless names, then there would be no reason why lists wouldn't be sentences. The sentence

(5) Native Dancer is a horse

[4] Cf. *BL*, p. 34 and *Translations from the Philosophical Writings of Gottlob Frege*. Edited by P. T. Geach and Max Black. Oxford: Basil Blackwell, 1960 (hereafter cited as '*G&B*'), p. 25.
[5] Cf. *G&B*, pp. 62-67.

stands for the True. And among its meaningful parts we can discern the predicate 'ξ is a horse', which stands for a concept. Now if a gapless name— 'the concept *horse*', say —could stand for the same concept, then

(6) Native Dancer the concept *horse*

should combine references, just as (5) does, and so stand for the True. But then (6) must be a proper name (specifically, a name of the True). But yet (6) arises by juxtaposing two (gapless) names, two type-1 names, and so, according to the theory of syntax, is not a proper name.[6] In order to make his theories of syntax and meaning consistent, then, Frege holds that meaningful expressions of different types cannot stand for (or express) the same thing.

Now an easy way for Frege to hold that function-names and proper names cannot stand for the same thing is to hold that functions (i.e. the references of function-names) and objects (i.e. the references of proper names) are different in kind. But he cannot stop here; he must hold that *no* two names of different types stand for the same thing. Thus:

> Functions of two arguments are just as fundamentally different from functions of one argument as the latter are from objects, (*BL*, p. 73) and
> second-level concepts . . . are essentially different from first-level concepts. (*G&B*, p. 50).

This difference in objects and various functions Frege puts metaphorically in terms of "incompleteness". And the difference is intimately bound up with the difference in the expressions which stand for them. This is reflected in:

> We see, too, that there are basically different types of

[6] Cf. Montgomery Furth, "Two Types of Denotation," *Studies in Logical Theory*: APQ Monograph Series, monograph no. 2 (Oxford, Basil Blackwell, 1968), pp. 19f.

functions, since the various argument-places are basically different. Those argument-places, in fact, that are appropriate for admission of proper names, cannot admit names of functions, and vice versa. Further, those argument-places that may admit names of first-level functions of one argument, are unsuited to admit names of first-level functions of two arguments. (*BL*, p. 77)

Frege is forced, then, by his theories of syntax and meaning, to hold that meaningful expressions of different types do not stand for the same thing. And to stress this, I think, he holds that there are different kinds of things, corresponding to the different types of meaningful expressions. And the difference he puts in terms of an incompleteness which is supposed to be analagous to the incompleteness of the names. Thus when Frege says that objects are saturated while functions are incomplete, the point emphasized is that proper names and function-names cannot replace one another *salve significatione*. Thus:

> The self-subsistence which I am claiming for number is not to be taken to mean that a number word signifies something when removed from the context of a sentence, but only to preclude the use of such words as predicates, or attributes, which appreciably alters their meaning;[7]

or, again,

> ... I call the function itself unsaturated or in need of completion, because its name must first be completed by the sign for an argument, in order to obtain a complete reference.[8]

[7] *The Foundations of Arithmetic.* Translated by J. L. Austin. Evanston: Northwestern University Press, 1968 (hereafter cited as '*FA*'), p. 72.

[8] This is from an unpublished manuscript of Frege's titled "Ausführungen über Sinn und Bedeutung" (dated 1891-1892) which is translated as an appendix to Montgomery Furth's thesis: *On Concept and Object.* Berkeley: University of California, 1964.

III

It is important to the Fregean theory of meaning that predicates are not syncategorematic, but stand for functions. And so it becomes a major drawback to the theory that it seems prevented from specifying what such a predicate stands for. That is,

(7) 'ξ is a horse', stands for $\triangle$.

cannot be made into a true sentence, no matter what name replaces '$\triangle$'. For Frege holds the following:

(8) Sentences stand for truth values.[9]
(9) Truth values are objects.[10]
(10) Only gapless expressions stand for objects.[11]
(11) No gapless expression stands for a function.[12]
(12) 'ξ is a horse' stands for a function.

And from (8)-(12) it can be shown that (7) cannot be made into a true sentence, no matter what name replaces '$\triangle$'. For if what replaces '$\triangle$' has gaps, then the result of the replacement has gaps, and so cannot stand for an object (by (10)), and so cannot stand for a truth value (by (9)), and so cannot be a sentence (by (8)), and so cannot be a true sentence. If, on the other hand, what replaces '$\triangle$' has no gaps, it cannot stand for a function (by (11)), and so cannot stand for what 'ξ is a horse' stands for (by (12)), and so the result of the replacement, although a sentence, cannot be a true sentence.[13] Thus although Frege wants to say that predicates do have reference, he is prevented from saying what their reference is.

[9] Cf. *G&B*, pp. 32, 63-65, and *BL*, pp. 35f.
[10] *BL*, p. 36.
[11] *Ibid.*
[12] *BL*, p. 34.
[13] This argument is that of Dummett's review of *G&B* (*Mind*, 1954), p. 102.

This, of course, is not news to Frege; he was aware that "by a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept". (*G&B*, p. 54). But, he seemed to think, this is not an important difficulty, just "an awkwardness of language". The solution of this awkwardness is for the reader to "meet me half-way . . . —not begrudge a pinch of salt". The problem seems to me much graver than this; and the solution, I think, is different.

It might be thought that Frege can circumvent this difficulty by insisting on his *Grundlagen* thesis that one should "never . . . ask for the meaning of a word in isolation, but only in the context of a sentence." (*FA*, p. X.) Dummett rightly says that this principle "has no place in Frege's later philosophy, since it accords a distinctive position to sentences which he was no longer prepared to recognize".[14] But then perhaps the thesis should be modified to: never ask for the meaning of a word in isolation, but only in the context of a proper name.

Recognizing every meaningful part of sentences as either a proper name or a function-name, this revised principle seems to have two parts: (i) proper names have sense and reference (in isolation), (ii) function-names, though they do not have sense and reference in isolation, contribute to the sense and reference of complex proper names in which they occur. The principle, so understood, can lead in at least two directions, neither of which, I think, is compatible with things Frege holds.

One way to understand (ii) is that function-names do not stand for or express at all, although they do contribute to the meanings of complex proper names in which they occur. In short, function names are syncategorematic. The explanations of the theory of meaning, as we saw earlier,

[14] "Frege, Gottlob," *Encyclopedia of Philosophy*. Edited by Paul Edwards. New York: Macmillan, 1967, Vol. 2, p. 233.

proceed by way of holding that no meaningful part of sentences is syncategorematic. This, I think, is so important to the theory, that to give it up is to give up the theory. Of course, this may be what is needed; but it can hardly be described as helping Frege out of a jam.

Another way of understanding (ii) is that function-names do not stand for or express in the same manner as do proper names.[15] And this fact becomes painfully apparent when we ask for the reference of a predicate in isolation and expect an answer similar in form to the answer in the case of proper names. In the case of proper names we both *ascribe* and *specify* reference. This often takes place in the forms:

(I) $\exists x\ A$ st x
(II) A St a

where 'St' is short for 'stands for'. (I) and (II) work perfectly well for proper names, but they do not work for function-names: for all completions of (7) have the form of (II), but none of them work. We must, then, find new forms for ascribing and specifying reference for function-names.

It is likely to be objected at this point that if we are departing from (I) and (II) as forms of ascription and specification of reference for function-names, if we admit that function-names do not stand for in the same manner as do proper names, then we should not ascribe reference to them at all. They stand to their "references" in some other relation than do proper names, and so we should really not say that they stand for anything, or that they have references. This is the same kind of objection which is brought against saying that sentences have references.[16] I think it is misplaced in the case of sentences, and I think, for the same reason, that it is misplaced here. The theory of meaning is (in part) a theory about how the meaningful parts of a sentence

[15] This is Furth's view (*loc. cit.*), to be examined below.
[16] Cf. Dummett, *loc. cit.*

contribute to the sentence's truth value; the objects named by proper names, and the functions associated with function-names are relevant to this contribution. And this is what justified calling them all, as well as truth values, 'references'; we should understand nothing more than this by the appellation.

The following comment, Furth thinks, hints as what forms should replace (I) and (II) for ascribing and specifying reference for function-names:

> It would be assumed here, as in all our considerations of this sort, that "$\Phi(\xi)$" always acquires a reference if in it we replace "ξ" by a name that stands for an object. Otherwise I should not call $\Phi(\xi)$ a function.[17]

The forms are the following (where C(B, a) is the completion of (the incomplete (gapful) expression) B with (the complete (gapless) expression) a):

(III) $VaVbVx\ (a \text{ St } x \ \&\ b \text{ St } x \rightarrow \exists y(C(B, a) \text{ St } y \ \&\ C(B, b) \text{ St } y))$

(V) $VaVx\ (a \text{ St } x \rightarrow C(B, a) \text{ St } F(x))$.

Here (III) is the form of ascription of reference to the function-name B, while (V) is the form of specification of the reference of B.

A function-name, on this account, does not have reference in virtue of bearing a relation to some entity, a function. Rather, it has reference in virtue of yielding referring names upon completion by referring names. One of the consequences of this is that function-names might have reference even though there were no functions; thus generalization of function-names will be unwarranted. Frege, of course, requires such generalization, and needs functions to carry out his reduction of arithmetic to logic. For this program to be served, then, it seems clear that function-names must have reference

[17] *BL*, p. 41; cf. also p. 84.

pretty much in the way proper names do, in a way that allows generalization. So this explanation of reference for function-names will not adequately serve that program.

IV

Let us look more closely at (7), then, and see, if we can, how Frege's difficulties arise. 'Stands for' (in (7)), it would be thought, stands for a binary-relation: a function which takes the predicate 'ξ is a horse' and a function to truth value. Such a relation would be, in Frege's terminology, "unequal-leveled".[18] We might think we could represent the situation as

(13) St ('ξ is a horse', $\Phi(\ )$),

but, of course, we can't: (13) contains a gap and thus cannot stand for a truth value, but yet it would were it the result of putting the right sorts of expressions in the argument places of a relation-word. This is just our original trouble with (7) all over again.

But this trouble seems to be one which is not confined to the relation of standing for; why shouldn't the difficulties with (13) be difficulties with second-level function-names generally? That is, all second-level functions take as arguments first-level functions. But this cannot be pictured as

(14) $M(\Phi(\ ))$,

no matter what the second-level function-name is. The difficulties with truly completing (7) would seem to be, equally, difficulties with making a true sentence out of *any* second-level function-name.

Frege, however, did not think that all second-level function-names are problematic. How, then, did he handle the unproblematic ones? Frege's most often used example of a

[18] Cf. *BL*, §22.

second-level function-name is the universal quantifier. The application of the universal quantifier to a first-level function-name doesn't look like (14), however; rather, it looks like:

(15) $Vx\,\Phi(x)$.

The universal quantifier carries with it bound variables; these stand in the argument places of the first-level function-name. Consequently, the first-level function-name does not stand by itself (gapfully), nor is it a gapless name. The solution of the difficulties, in the case of the universal quantifier doesn't take either horn of the dilemma of (7): the bound variables don't make the first-level function-name into a complete name, nor do they leave it to fend for itself.

This treatment by Frege of second-level function-names is not untypical, though there are such function-names which don't have bound variables.[19] An example Frege gives of an unequal-leveled function-name with bound variables is the first derivative of a function, evaluated at a point. This is not represented on the pattern of (14), that is, as

(16) $D(\Phi(\ \ ), \triangle)$,

but rather something like

(17) $\dfrac{d(\Phi(x))}{dx}(\triangle)$

or, perhaps, as

(18) $Dx\,(\Phi(x), \triangle)$.

Thus we are not left with an incomplete expression, as in (16), nor are we forced to fill the argument place in '$\Phi(\ \ )$'

[19] E.g. '$\varphi(2)$' and '——$\varphi(2)$'; cf. *BL*, p. 75. Cf. also p. 79 where Frege says, "We indicate a second-level function of one argument ... in this way: "$M\beta(\varphi(\beta))$"."

with the name of an argument. Here too the middle road of filling the gap with a bound variable has been taken. Thus

> The first derivative is accordingly to be regarded as a function of two arguments, the first of which must be a first-level function of one argument, the second of which must be an object. (*BL*, p. 75).

This suggests that we might treat standing for as an unequal-leveled function in the same manner. We should not, then, have represented things as (13) (which is on the pattern of (16)), but rather with bound variables (on the pattern of (18)), thus:

(19) St x('ξ is a horse', $\Phi(x)$).

Here, as with the universal quantifier and the first derivative, the middle road has been taken of filling the argument place in the first-level function-name with a bound variable; thus, again, both horns of the dilemma of (7) have been avoided.

Initial reaction to this suggestion, I think, is that it is entirely *ad hoc.* It is not at all clear what role the bound variable is supposed to be playing in (19), nor that it is needed. Bound variables don't seem to be at home in (19) as they are in (15) or (17). This reaction, I think, is wrong. Though it doesn't seem very clear that bound variables are required, or called for, in (19), neither, I think, does it seem very clear that they are required, or called for, in (15) or (17). Why should 'x's be needed in (17)? Wouldn't (16) serve as well? Is it just a historical accident, then, that mathematicians use (17) instead of (16) when differentiating? I think not. Although when the function is monadic, we could get along just as well with (16), this is not so when the function is polyadic ("is a function of" two or more "variables", as they say). In these cases the bound variable is needed to tell which "variable" the function is being differentiated with respect to. Similarly, in (15), little confusion would result from dropping the bound variables thus:

$$V(\Phi(\ ))\text{.}$$

Confusion would, though, be the order of the day in more complex sentences, in which there were several quantifiers:

$$V\ \exists(\Psi(\ ,\ ))$$

is, at best, ambiguous. Here bound variables are required to keep the references straight.[20] *So too, I think, with standing for.* The bound variables in (19) appears pointless just because the function-name involve has only one argument place. But, just as with quantification and differentiation, if the function-name had several argument places, then bound variables would be needed to keep track of the references. Thus if

St x,y ('ξ is bigger than ζ', $\Psi(x,y)$)

is the True, then

St x,y ('ξ is bigger than ζ', $\Psi(y,x)$)

will be the False: *the bound variables here keep the references straight.* This is similar to the diference between

$$Vx\ \exists y\ Rxy$$

and

$$Vy\ \exists x\ Rxy.$$

And just as the variables keep track of reference in

$$(\iota x)\ (x \text{ is biggest}) = (\iota x)\ (\neg \exists y \Psi(y,x)),$$

[20] This is not to say that we couldn't do without variables à la Schönfinkel; but variables are necessary in this setting.

so do they keep track of reference in

St x('ξ is biggest', $\neg \exists y \Psi(y,x)$).[21]

(7), then, should be transformed and completed as in (19):

St x('ξ is a horse', x is a horse),

thus resulting in a true sentence. And other higher level function-names —like 'is a (first-level) concept'— also carry along with them bound variables. Thus:

FC x(x is a horse).

And, in similar fashion,

St Φ ('$Vx\ \varphi(x)$', $Vx\ \varphi(x)$)

and

SC Φ ($Vx\ \varphi(x)$),

One of these higher level function-names will be a sign for identity between (first-level) functions:

$= x(\varphi(x),\ \Psi(x))$.

Given what Frege says (*G&B*, p. 80), this is likely the same second-level relation as

$Vx\ (\varphi(x) = \Psi(x))$,

or, in the terminology of courses-of-values,

$\grave{\epsilon}\,\varphi(\epsilon) = \grave{\epsilon}\,\Psi\,(\epsilon)$

[21] The situation is the same for notation for courses-of-values; cf. *BL*, §§9f, 36.

(these are equivalents by basic law (V), *BL* §§, 20).[22] This is, of course, a different relation than that of identity among objects, for the one is second-level while the other is first-level; it is this latter relation for which Frege reserves the title 'identity'.

What of proper names? Presumably there is an unproblematic relation holding between proper names and their bearers. The situation here, perhaps may be pictured as:

(20) St ('Aristotle', Aristotle).

This 'St' stands for a different relation than does the 'St' in (19): in the case of (19) the relation is unequal-leveled, while in (20) it is first-level. Thus, perhaps, there are, after all, "two types of denotation"; but, still, this should not lead us to the objection countenanced above (p. 9).

We should notice that Furth's suggestion embodies our basic insight: standing for (for first-level function-names of one argument place) must be an unequal-leveled relation holding between a predicate and a function, and it should carry bound variables to fill the argument places of the name of the function. (V) provides just that: here we have a complex function-name with two argument places one of which is filled with a name of a predicate —namely 'B'— and the other of which is filled with a name of a function —namely 'F'— and the argument place of the function-name is filled with a bound variable. (V), then, has the form of (19). Furth has attempted to obtain a satisfactory unequal-leveled relation through appeal to the first-level relation of standing for which holds between proper names and their bearers. I have urged that Furth's proposal has defects serious enough to discredit it. A relation which does the jobs Frege wants done, I think, cannot be obtained definitionally; thus it must be accepted as primitive.

[22] This is also strongly suggested in "Ausführungen über Sinn und Bedeutung."

RESUMEN

La teoría de Frege es un intento por dar respuesta a los siguientes problemas: (1) ¿De qué manera las partes significativas de una oración contribuyen al valor de verdad de ésta? (2) ¿De qué forma, las partes significativas de una oración contribuyen al pensamiento que expresa la oración? Para Frege, antes de obtener una solución a (1) y (2) se debe responder a las siguientes cuestiones: (3) ¿Cuáles son las partes significativas de la oración? y (4) ¿Qué combinaciones de las partes de la oración constituyen oraciones? Las respuestas de Frege a (3) y (4) abarcan lo que llamaré su teoría sintáctica, y las soluciones a (1) y (2) comprenden su teoría del significado. En este artículo se expone un resumen de estas tesis y se ofrece una solución a la dificultad que constituye el hecho de que resulta imposible, en principio, especificar la referencia de un nombre de función.

La teoría sintáctica de Frege distingue dos clases de partes significativas en la oración: los nombres de función y los nombres propios. Las primeras expresiones tienen funciones por referencia y sentido. Los nombres propios poseen objetos como su sentido y referencia. Las funciones toman argumentos y dan por resultado valores. Por su parte, los nombres de función tienen lugares para ser ocupados por los nombres de los argumentos; estos lugares se indican con espacio. Las funciones y los nombres de función tienen la característica de pertenecer a un nivel. Así una función que toma como argumentos las referencias o los sentidos de nombres propios, es una función de primer nivel; una función que tiene por argumentos funciones de primer nivel, es una función de segundo nivel y así sucesivamente. A aquellas funciones de primer nivel con un argumento, cuyos valores son los valores de verdad, se les denomina "conceptos". Los cuantificadores son nombres de funciones de segundo nivel y sus referencias son funciones de segundo nivel. Existen también nombres de funciones de nivel desigual, esto es, expresiones con más de un lugar de argumentos que son ocupados con nombres o nombres de función de distintos niveles. Los lugares de argumento se indican con espacios que son ocupados por letras consonantes griegas. Estas letras tienen la doble función de mantener, por una parte, disponible el lugar del argumento, y por otra, indican la forma adecuada de completar los nombres de función. En algunos casos, cuando el argumento apro-

piado es el nombre de una función de primer nivel, los huecos de ella, esto es, sus propios lugares de argumento se ocupan por variables ligadas con la función de segundo nivel.

Con estos elementos, Frege establece que cada oración está constituida por un nombre de función principal, cuyos lugares de argumentos están ocupados con expresiones de la clase apropiada. Con esto se da solución a los problemas en la teoría sintáctica. Por lo que toca a la teoría del significado, el valor de las oraciones será el valor que adquiere la función, cuando toma sus argumentos adecuados. De la misma forma, se combinan los sentidos de las partes significativas para dar origen al sentido de la oración. A partir de esta solución, se desprenden dos principios de sustitutividad, uno para las referencias y otro para los sentidos de las expresiones.

Para la teoría del significado, los predicados pretenden representar a funciones. Pero parece ser que precisamente la teoría no puede especificar aquello que un predicade representa. Debido a un argumento desarrollado por Dummett, se establece que aunque Frege desea expresar que estas expresiones tienen referencia, está imposibilitado de decir qué cosa es su referencia. La dificultad se expresa en el hecho de que es imposible obtener una oración verdadera a partir de la expresión :" 'ξ es un caballo' representa a $\triangle$", donde $\triangle$ representa a la referencia de la expresión "ξ es un caballo".

Ante esta dificultad, una línea de solución podría estar en la conocida tésis: "nunca debe preguntarse por el significado de una palabra en aislado, sino sólo en el contexto de una oración". Dado que esta afirmación no reaparece en la filosofía posterior de Frege, debido a cambios en la manera de considerar a las oraciones, esta tesis, podría modificarse de la siguiente manera: "nunca debe preguntarse por una palabra aislada, sino sólo el contexto de un nombre propio". Si se observa esta versión, el principio parece constar de dos partes : (i) los nombres propios tienen sentido y referencia en aislado; (ii) los nombres de funciones, aunque no tienen sentido y referencia en aislado contribuyen a estos dos aspectos de los nombres propios complejos. Sobre esta base, la tesis podría dar origen a dos direcciones de solución, pero ninguna de ellas resultaría compatible con otras afirmaciones de la teoría. La primera de ellas es interpretar que los nombres de funciones no representan, ni expresan en absoluto, aunque aportan al significado de los nombres propios complejos. Pero ello equivale a decir que estas expresiones son sincategoremáticas y esto contradice lo establecido dentro de la teoría del significado. Otra forma de considerar (ii) es interpretar el que los nombres de funciones no representan, ni expresan de la misma manera que lo hacen los nombres propios. Dentro de este trazo general, se puede analizar la expresión " 'ξ es un caballo' representa a $\triangle$ ", donde "representa α", y señalar que indica a una función binaria de segundo nivel, cuyos argumentos son el 'ξ es un caballo' y una función de primer nivel. Pero también aquí se reproduce el problema de que es imposible obtener una oración verdadera a partir de esta función de segundo nivel. Sin embargo, Frege puede obtener oraciones verdaderas a partir de funciones semejantes, como es el caso del cuantificador universal. Y lo puede hacer haciendo uso de las variables ligadas. Si se enfoca la dificultad específica de la función: "Representa ("ξ es un caballo", ϕ ( )) como un problema general de las funciones de segundo nivel, entonces en esto se puede hacer uso de expediente de las variables ligadas.

De esta manera los nombres de funciones de primer nivel no representan por sí mismas, pero tampoco quedan incompletos. Las dificultades de esta solución están en considerar la legitimidad del papel que desempeñan las variables ligadas en el caso específico de la función que se ha estudiado.