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EXTENSIVE MAGNITUDES: METAPHYSICS, REPRESENTATION AND EPISTEMOLOGY

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The metaphysical nature of A is *explanatorily prior* to the nature of B in the order of philosophical explanation if and only if the true account of what individuates B invokes A but not *vice versa*. A general thesis is that the true metaphysics of every domain is explanatorily prior to the nature of the representational contents that represent that domain. In his book *The Primacy of Metaphysics*, Christopher Peacocke argues at length that metaphysics enjoys such priority in the cases of extensive magnitudes, numbers, time and the self. Peacocke's book and the talks and seminars that preceded its publication are the inspiration for this special issue.

Extensive magnitudes are ones of types like area and distance that have parts and are subject to a natural (mereological) addition operation. *Intensive* magnitudes are ones of types like frequency, temperature and speed that do not have parts and are not subject to such an operation. An attractive form of realism about extensive magnitudes of various types and levels —some more fundamental, some less— holds, first, that fundamental physical magnitudes are explanatorily prior to higher-level magnitudes and, second, that the metaphysical nature of extensive magnitudes of any level is explanatorily prior to the representational contents that represent them. It is arguable that some types of intensive magnitudes are subject to the second thesis and some are not. For example, temperature and relations among temperatures might be thought to be explanatorily prior to concepts such as those of heat and cold and of being hotter and colder. In contrast, if one combines the Lockean metaphysical view that being loud consists of standing in a relation to the property of sounding loud, with the view that any account of sounding loud must make ineliminable use of the concept *loud*, then subjective loudness is a case of *no priority*: metaphysics and representational content are each involved in the explanation of the other.[1]

Peacocke argues that extensive magnitudes of various types and levels play an ineliminable role in causal and explanatory laws, including the psychological laws governing the representation of magnitudes. This kind of realism appeals to a general mathematical theory of extensive magnitudes the truth of which requires that there are magnitudes that are not actually instantiated.[2] These, in addition to ones that are actually instantiated, are claimed to feature in causal explanations and explanatory laws. So this is a kind of realism for which the too often misapplied term "Platonism" might not be so misleading. Magnitudes, according to this kind of realism, are *not* examples of the kinds of abstract entities that we find in modern mathematics.

In the present special issue the aforementioned form of realism comes under considerable scrutiny and pressure. Geoff Lee, in his extremely rich paper "Against Magnitude Realism", draws on the first thesis that fundamental physical magnitudes are explanatorily prior to higher-level magnitudes to argue in favor of a reductionist account of higher-level magnitudes. In the second paper, "Explanatory Problems for Mass Additivity and Dynamics", Zee Perry offers an ingenious argument that taking the phenomena of mass summation —that the magnitudes of the parts of a composite entity sum to the magnitude of the whole— to be metaphysically independent of dynamical laws governing massive bodies results in unexplained relations between the summed masses of composites and the dynamics of massive bodies. Perry then develops this explanatory problem into an objection to realism about extensive magnitudes. Both Lee's and Perry's papers are important contributions to the growing literature on the metaphysics of magnitudes that demand a response.

Turning from metaphysics to epistemology, there are various epistemological options for the realist. As already noted, according to the kind of realism under discussion, extensive magnitudes play causal and explanatory roles in laws connecting magnitudes to other magnitudes and to perceptual representational states. As a result, magnitudes are in Peacocke's phrase "empirically detectable" (2015). Further, while realism presupposes the aforementioned general mathematical theory of magnitudes, it might be argued that this theory

[1] Peacocke 2019, pp. 19–20.

[2] See Scott 1967, which is based on *Elements* Book V, usually attributed to Eudoxus.

enjoys empirical justification due to its role in an empirical explanatory theory. Alternatively, it can be argued that the general theory is known *a priori* and requires at least a moderate rationalist epistemology according to which it must be *understood* —albeit in a way that draws on our intuitive grasp of magnitudes. For example, one must understand certain definitions and axioms that extensive magnitudes satisfy in virtue of being extensive magnitudes. These are definitions and axioms that state what it is to be such a magnitude and whose consequences, arguably, are known *a priori*. For example, extensive magnitudes are subject to a (mereological) summation operation and consequently stand in ratio relations that can be compared and have a criterion of equality.

Turning from comparison of magnitudes to their measurement, realism about extensive magnitudes also requires a formal theory of measurement that is interpreted in terms of certain function: a structure preserving mapping between represented relations among extensive magnitudes and representing relations among numbers.[3] Further, while it is debatable whether the general theory of magnitudes has an empiricist or a rationalist epistemology, it is natural to think that the formal theory of their measurement has an empiricist one. This is because the represented relations might be thought, and have been thought, to hold among material items that have magnitudes rather than among the magnitudes themselves.[4] By giving the formal theory an empirically observed material interpretation, and then formally deriving numerical results about this interpretation from the axioms of the theory, one might hope to place measurement on a formal and empirical foundation.

If the reasoning in the last two paragraphs were headed in the right direction, then we might expect to have the following epistemological options: either a rationalist foundation for, or a more coherentist and empiricist justification of, the general mathematical theory of magnitudes, together with an empiricist foundation for measurement. Unsurprisingly perhaps, things are not so straightforward. In "The Philosophical Significance of the Representational Theory of Measurement", Jo Wolff points out that the aforementioned kind of empiricist-foundationalism for measurement faces various insuperable problems.[5] Rather than accepting as an alternative the view that

[3] See Peacocke 2015.

[4] See the references to work on the formal theory of measurement in Jo Wolff's paper.

[5] See also the objections in Peacocke 2015, 2019.

theories of measurement are merely formal theories that characterize different possible measurement structures, Wolff argues that they provide a "semantic foundation" for measurement because they tell us how to use number-involving relational structures as representations: more precisely, as measurement-representations of extensive-magnitude-involving relational structures.

At this point an objection can be made to Wolff's proposal from the realist perspective described earlier —an objection to which Wolff has a response. A relational structure is an example of the kind of abstract entity that we find in modern mathematics. But, as I said earlier, from the realist perspective magnitudes are *not* examples of the kinds of abstract entities that we find in modern mathematics. Wolff puts the point more neutrally. The representational account of formal theories of measurement only tells us how to represent one mathematical structure with another but does not tell us how to represent magnitudes in the empirical world (where, the realist would add, they play a causal and explanatory role). Peacocke's realist solution is that the variables in the axioms of a formal theory of measurement range over magnitudes not over material items that have such magnitudes.[6] So interpreted, formal theories of measurement represent relations among magnitudes, conceived realistically, and so take on metaphysical significance. Obviously, this is controversial and should be evaluated in the light of Wolff's more modest proposal.

Magnitudes can be perceived as well as measured. According to Peacocke, extensive magnitudes play an ineliminable role in the explanatory psychological laws governing perceptual representation of them —a special case of his more general thesis that they play an ineliminable role in causal and explanatory laws. In the case of perception, the basic idea is that analog representation is representation of magnitudes by magnitudes, where the representing magnitude does not digitize the represented one. But what exactly is required for representation without digitization? Following Nelson Goodman,[7] let's first draw the analog/digital distinction for representational *vehicles* (symbols) as follows. A representational vehicle is digital, relative to the subject's perceptual and recognitional capacities (including memory), if and only if these vehicles enable one to recognize and re-identify them on the basis of those capacities. Otherwise they are analog. For example, the following symbols are digital:

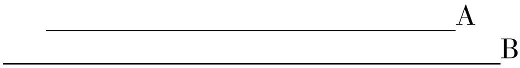
[6] Peacocke 2015.
[7] Goodman 1968.

| | |

1 meter

π.

Analog representational vehicles contrast with digital ones in this respect; for example, relative to our capacities, the following representation of the magnitudes A and B and their ratio is plausibly analogue:



Similarly, the *contents of perceptual representations* are digital, relative to the subject's perceptual and recognitional capacities, if and only if these contents are enabled by perceptual recognition or re-identification of their referents.[8] It follows that perceptual representations of magnitudes are not digital, but analog, since they are not enabled by perceptual recognition or re-identification. Notice that according to this account, the metaphysical nature of magnitudes is explanatorily prior to that of the representational contents that represent them. When we are not able to perceptually recognize or re-identify the magnitudes we perceive, in which case we represent them with analog contents, this is explained by the fact that the magnitudes are more fine grained than are our capacities to recognize or re-identify them.

Peacocke's account is consistent with the view that analog representations mirror (are isomorphic to, or bear some other structure-preserving mapping towards) what they represent:

> a structure-preserving relation, such as an isomorphism, is to be expected when there is representation by instances of some magnitude-type. Different magnitudes of the given magnitude-type will represent instances of some given magnitude or property in the world. Different values of the representing magnitude will correspond systematically to variations in the represented magnitude or property. So comparative closeness relations between instances of the representing magnitude will map on to corresponding closeness relations between what is represented. (2019, p. 57)

[8] This is very close to Peacocke's account. See Peacocke 2019, p. 65.

The account can explain among other things: perception of magnitudes with analog perceptual content, how representations of magnitudes are drawn on by perceptual representations of shapes; the perception of the *ratio* of two magnitudes (such as perception of an octave increase in pitch) in terms of perception and comparison of magnitudes; and, more generally, how analog computation on representing magnitudes generates further representing magnitudes.

In his paper "Contents and Vehicles in Analog Perception", Jacob Beck argues that while Peacocke's account is on the right track, if it is to serve its purpose of explaining real patterns in our perceptual capacities, the account needs to recognize that the contents of perceptual representations are analog in virtue of there being *gradual change* in our ability to perceptually recognize or re-identify the magnitudes that we perceive. Beck then goes on to argue that the best explanation of such gradual change, and so of the *contents* of perceptual representations being analog in his amended sense, is that the *vehicles* of perceptual representations are analog in the *mirroring sense* described above.

One of the most lucid and persuasive defenders of the mirroring account of analog representation is Corey Maley, whose paper "Icons, Magnitudes, and their Parts" is the last in this special issue. In this paper, Maley expands considerably on his previous theory and on the notion of structure preservation to which it appeals. He also argues that while, as I noted earlier, extensive magnitudes are subject to a mereological summation operation and while, as a result, representations of extensive magnitudes by extensive magnitudes satisfy the much discussed parts principle —that the parts of an analog representation of A are representations of the parts of A—, the significance of this has been overstated. To make his case, Maley draws on examples of analog representation in which the whole representing entity is a sum of smaller magnitudes that are not parts of the whole in the mereological sense, because we can restrict different dimensions of variation in ways that result in a partitioning of the representing entity that is not mereological. To this extent, it complements proposals due to Peacocke, Sam Clarke and John Kulvicki.[9] The contributions of Beck and Maley will be of great interest to anyone trying to understand not only how to draw the analog/digital distinction but also how to draw it so that it is of relevance to cognitive science.

It remains to connect the contributions in this special issue with broader epistemological concerns such as the epistemology of the

[9] Clarke 2022, Kulvicki 2015, Peacocke 2019.

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general theory of magnitudes —the theory that mathematically characterizes the extensive magnitudes that are the subject of this issue. I will close with a tentative suggestion about how to connect the perception of magnitudes with a rationalist theory of understanding for the general theory of magnitudes.

In the case of perceptual analog representation, the magnitudes represented are more fine-grained than our capacities to recognize or re-identify them. Analog representations represent magnitudes perceptually with only *approximate accuracy*. If so, then all we should require is that *some* structure is mirrored when we represent magnitudes perceptually, i.e., that structure is preserved only up to a certain fineness of grain. How might magnitudes be represented more accurately? One thing we might say is that the role of analog representation of magnitudes and ratios is to get these into the representational picture before *other* operations are performed, in order to represent magnitudes and their ratios more precisely. On this view, once reference to magnitudes is secured by perception, our ability to represent them can then be *enhanced*, with unlimited precision, by performing different computations on representations of the same entities —representations that Beck, Maley and Peacocke might acknowledge to be digital. According to this proposal, it is not that there are two kinds of magnitude —one represented perceptually and another that can only be represented by the general mathematical theory of magnitudes. Rather, there is one kind of magnitude that is first represented in a perceptually analog and imprecise way, then represented digitally, and then represented with greater accuracy by performing computations on these digital representations. It will be a worthwhile exercise, then, to try and construct an account of the capacities required to do this —one that keeps front and center the perceptual mental content discussed by Peacocke, Beck and Maley.

One option in the space of options for the realist, who recognizes the role of magnitudes in perception, is to draw on recent work by Tyler Burge on the relationship between perception and perceptually based belief, on the one hand, and more general context-independent thought, of the kind that we are now trying to explain, on the other:

> The problem is to explain what it is to separate attribution from its role in aiding singular reference [e.g. that square object], to arrive at propositional predication [e.g. that object is square]. A capacity for such separation is a central aspect of achieving the specific context independence and generality that are embodied in pure attribution,

> propositional thought, and rational inference. (2010, p. 539 [examples added].)
>
> In thought [. . .] we commonly make occurrent use of attributives that do not guide a contextual singular application in singling out a referent [. . .] For example, the following occurrences of attributives in thought do not guide purported context-bound reference to a particular: cats and are animals in cats are animals; plants and are green in some plants are green; is a number in 3 is a number. (2010, p. 541)
>
> Conceptual attributives necessarily have some uses, contributions to veridicality conditions, where their representational function is *not* to guide referential application. (2022, p. 676)

A natural idea, then, is to begin with our capacities to perceive and compare magnitudes in an analog fashion and to isolate the role of what Burge calls "perceptual attributives", the general elements of perceptual content that represent particulars as being of kinds, having properties or entering into relations —as in when we perceive a plant as green or an object as cubical. In particular, while pursuing this option, we are interested in the role of magnitude-attributives, the general elements of perceptual content that represent, say, bodies and surfaces as having length or distance, and which occur in "here and now" contextually bound thoughts such as this distance is too much to traverse by jumping or this angle is too obtuse to rotate comfortably. (Henceforth, I adopt Burge's convention of underlining reference to propositional representational contents.) There are also comparative relational perceptual magnitude-attributives, such as the ones that are attributed when we think contextually bound thoughts such as this body is larger than that body, or this note is an octave up from that note.[10]

The next step is to claim that such attributives are sometimes separated from such contextually bound thoughts, so they can play their role in what Burge calls "pure attribution", "pure predication", or "conceptual attribution": "Unlike perceptual attributives, conceptual attributives function constitutively to contribute to propositional structure" (2022, p. 713).

Attributives certainly seem to have this role in some plants are green, as they also seem to in some American cities are a considerable distance apart and the (partly) relational thought that some people are taller than other people.[11]

[10] See Burge 2022, p. 177.

[11] It is important to note that this claim that perceptual attributives are separated does not concern the developmental stage of any individual, since we might be

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If this is correct for the case of magnitudes, then we have a capacity for context independent attribution of magnitudes of various types, which is a prerequisite for propositional thought involving attribution of those types. Drawing on this and other capacities, we can now engage in context-bound perceptual attribution of magnitude types, thinking thoughts such as that the length of this table makes it a perfect fit for this nook and the comparative relational thought that the length of this table is less than twice its width.

Now we can say that we not only have capacities constitutive of pure (as well as perceptual) attribution, and so propositional thought involving attribution of magnitudes of various types, but also have analogous capacities concerning magnitude itself. In which case, we can think the thought that length and width are both types of magnitudes, and so attribute the concept magnitude to each of these types.

Next, according to this proposal, we can introduce a *mental name* for a magnitude, A, and associate it with an iconic analog representation of a magnitude such as a line. This iconic analog representation will satisfy the parts principle mentioned earlier. Further, recalling that multitude is a type of magnitude, we can accumulate a certain multitude of copies of A and compare A with the resulting multiple of A. Furthermore, recalling that we can separate from perceptual attribution the pure attribution of individuals entering into relations, we can separate the pure attributive is less than, associate it with the symbol $<$, and think thoughts such as

1. $k\mathrm{A} < m\mathrm{B}$
2. $k\mathrm{B} < m\mathrm{A}$.

We can then check whether such inequalities are satisfied and so represent magnitudes and ratios with unlimited precision. This is just what the general theory of magnitudes requires us to do.

Of course, there are the further questions of why the general theory has the axioms and definitions that it does and what was required to discover them. Answering these would take us into the history and philosophy of mathematics and beyond the scope of this introduction. My aim is simply to connect recent work on perceptual

born with the capacities for both perceptual and pure attribution; rather, the claim concerns kinds of objective representation and says, specifically, that those who can think propositional thoughts must have capacities that are less dependent on their particular context than are their perceptual capacities.

representation of magnitudes with a suggestion about how we are able to engage in the kind of reasoning about magnitudes that is required for discovering and understanding the general theory —the theory that mathematically characterizes the extensive magnitudes that are the subject of this special issue.[12]

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