

OUTLINE OF A DIALOGICAL THEORY OF TRUTH

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SUMMARY: In this essay I propose two theories of truth and show how they deal with semantic paradoxes. Their most salient feature is that they are based on a game-theoretic understanding of logic and meaning. “Truth”, therefore, is understood dialogically, as agreement between parts. I compare this proposal with a similar one already existing in the literature —Dutilh Novaes and French 2018—, and highlight the advantages of mine. The theories of truth I present are non-trivial, substructural (in a sense to be clarified) and capture strong intuitions about truth. Some philosophical recommendations in favor of our understanding of Dialogics are delivered along the way.

KEY WORDS: substructural logics, semantic paradoxes, dialogics, non-transitivity, non-reflexivity

RESUMEN: En este ensayo ofrezco dos teorías de la verdad y muestro cómo éstas manipulan las paradojas semánticas. El aspecto más sobresaliente de estas teorías es que se basan en una comprensión de teoría de juegos de la lógica y la semántica. La “verdad”, por tanto, es entendida de manera dialógica, como un acuerdo entre partes. Comparo esta propuesta con otra similar que ya existe en la literatura —Dutilh Novaes y French 2018—, y señalo las ventajas de la mía. Las teorías de la verdad que presento son no-triviales, subestructurales (en un sentido a ser clarificado) y capturan fuertes intuiciones acerca de la verdad. También se ofrecen algunas recomendaciones filosóficas en favor de nuestra forma de comprender la Dialógica.

PALABRAS CLAVE: lógicas subestructurales, paradojas semánticas, dialógica, no-transitividad, no-reflexividad

1. *The Problem*

It may be said that when Pilate asked, “Are you the king of the Jews?” and Jesus replied “you say that I am” (*Lucas XXIII, 3*), what was at stake was not the truth of the sayings about Jesus but their agreement about these. There is a tradition of logicians that see formal validity as a matter of such agreements. They are the dialogue logicians or *dialogicians*.

Dialogicians have developed a broad number of investigations along the last decades, and many subjects and problems treated by other logicians have been also explored by them (Clerbout and McConaughey 2022). Still, one such problem that is severely underdeveloped in its dialogical trend is the solution to semantic paradoxes. The purpose of this paper is to fill in this gap by presenting two dialogical truth theories which are non trivial and substructural, in a sense to be specified.

One may ask why it is that dialogicians have payed little attention to semantic paradoxes. A plausible explanation is that the key concept among the latter, “truth”, has an unclear dialogical meaning. As in all game-theoretic traditions, in this trend validity is captured by the existence of winning strategies; but unlike proposals such as Hintikka’s, in Dialogics one does *not* have a model in sight. Therefore, there is not a straightforward way to relate winning strategies to truth-preservation.

Yet it seems reasonable to expect that words like “truth” were amenable to a dialogical explanation, because we use such words in language games. My bet is that for some of these games it makes sense to think of dialogical truth in the following terms:

Dialogical Truth: to claim that a proposition is true amounts to declare oneself as being able to defend that claim.

The following well-known observation about truth may help to illustrate the adequacy of this definition:¹ in most contexts, when making a claim about anything, it is redundant to add the predicate “. . . is true” to that claim. And it may be argued that this is *because* to claim that a proposition is true amounts to claim that proposition. So, from a model-theoretic point of view, the propositions “The sky is blue” and

“The sky is blue” is true

Will come out true in the exact same cases (models). And from a dialogical point of view, someone should be able to defend the claim that

I can defend the claim “The sky is blue”

In the exact same cases in which she can defend the claim “the sky is blue”. So it looks like a plausible suggestion to state that both

¹ Thanks to an anonymous referee for suggesting this.

notions, i.e., model-theoretic truth and dialogical ability-to-defend a claim, are two different explications of the same informal notion (truth).

An understanding of truth along these lines is useful for two reasons. The first is that it is at even with the Principle of Equality,² because it states the conditions of truth in terms of challenges and defenses of claims. The second is that it is amenable to exhibit paradoxical features. Consider the ordinary statement made by Chadwick:³

- (1) Whatever Piñera has said about the revolt of October 2019 is true.

Say Chadwick is facing a trial for his responsibility in the management of the revolt. The role of the public attorney is to challenge this stance and so Chadwick will have to defend it, and this defense will depend on the sayings of the president. Now suppose that the only thing Piñera had ever said about the revolt is:

- (2) Whatever Chadwick has said about the revolt of October 2019 is false.

If we understand “true” according to the proposed definition, then (1) expresses solely the commitment of Chadwick to defend (2); and (2) expresses the commitment of Piñera to defend that Chadwick cannot defend (2).

As it is easy to see, a defense of (2) amounts to show that (1) can be challenged. Thus a dialogical version of the yes-no paradox obtains: to successfully defend (1), Chadwick needs to successfully defend (2); but, to successfully defend (2), Chadwick must not be able to successfully defend (1)... and so on.

The truth theory that I will introduce in section 3 will capture this sense of dialogical truth, leaving open place for paradoxes like the one from this example. Then, two solutions with classical recapture results will be given. But first, in section 2 I will comment on one antecedent to my proposal and why it falls short as the dialogical truth theory I am looking for. A brief comparison between the two is given at the end of section 3. A final section recapitulates the conclusions.

² See section 3.

³ Former prime minister and cousin of Piñera, the Chilean president from 2018 to 2021.

2. *Previous Proposals*

Thus far the only dialogical approach to the semantic paradoxes (that I know of) is Dutilh Novaes and French 2018. There are two features of this proposal that are worthy of attention. First, we will look at the dialogical system itself. The authors say about this system that it

borrows elements from previous dialogical approaches in logic (in particular Lorenzen’s dialogical logic (Lorenzen and Lorenz 1978; Keiff 2009) and Hintikka’s game-theoretical semantics (Hintikka and Sandu 1997)) but it differs significantly from these earlier proposals in many respects. (Dutilh Novaes and French 2018, p. 131)

These “significant differences” will constitute severe drawbacks for the kind of truth theory we are looking for and will be our main reason to depart from this framework. Second, we will focus on the solution to the semantic paradoxes they endorse and the general conclusions they draw about the family of the so-called *substructural* solutions. As we will see at the end of the next section, our proposal will also disagree with their final diagnose on the paradoxes.

The common feature in all game-theoretical approaches to logic is that arguments are depicted as games between two players. Validity (and invalidity) is captured by the notion of *winning strategy*: the possibility to play in such an intelligent way that victory is attained no matter what the other does. The dialogical interpretation of logic defended by Dutilh Novaes (2013, 2015a, 2015b) considers dialogues between Prover and Skeptic. Prover has to *explain* to Skeptic that the conclusion follows from the premises, while Skeptic must *resist* to this conclusion as much as reasonably possible. So their dialogues are cooperative in some respects and adversarial in others.

One salient feature of this proposal is that it pays “no special attention to logical connectives; instead, [it focuses] on very general features of logical reasoning and argumentation” (Dutilh Novaes and French 2018, p. 131). In other words, this is a *dialogical interpretation* of a proof-theoretical system. A closer look on the details may help to better illustrate this claim.

The logical system used by Dutilh Novaes and French is a “sequent-to-sequent style natural deduction system” Dutilh Novaes and French (2018) introduced in Sørensen and Urzyczyn (2006), enriched with a paradoxical operator from Read (2000). From the point of view of pure logic, the system behaves just like a proof-theoretic device: there are rules of introduction and elimination, derivations take the

form of oriented graphs, and so on. Prover and Skeptic are called to justify such elements, as their interaction will provide the required information to produce valid derivations.

The game proceeds as follows:

Our dialogues begin with Prover offering a sequent, and play proceeds with Prover and Skeptic making alternating moves. A dialogue is won by Prover whenever the only sequents which Skeptic has available to challenge are instances of (Id).⁴ To see why this is the case, note that in any situation where Prover can do this, then all the formulas involved must have, at some earlier stage, appeared in a Skeptic offer, and thus have been granted by Skeptic. (Dutilh Novaes and French 2018, p. 137)

And the moves of each player are expected to be the following:

- A *Skeptic* move involves them challenging a sequent previously offered by Prover, offering a sequent that entails the challenged sequent, which they claim would convince them of the validity of the sequent challenged. (Dutilh Novaes and French 2018, p. 136)
- A *Prover* move involves them offering a set of sequents which they claim entail the sequent challenged by Skeptic on their previous move. This is achieved either because the set of sequents entail the sequent offered up by Skeptic in their previous move, or by a demonstration that a subset of the premises of the sequent offered up by Skeptic are inconsistent. (Dutilh Novaes and French 2018, p. 137)

Here's the example they present in pages 137–138 (slight notational changes):

Prover (1): I reckon that $A \supset C$ follows from $A \supset B$ and $B \supset C$.

Skeptic (1): Yeah? Well if that's so then suppose I grant you $A \supset B$ and $B \supset C$ along with A , how are you meant to get C ?

Prover (2): If you grant me B I can get C from $B \supset C$ (which you just granted).

Skeptic (2): But why should I grant you B ?

Prover (3): Well if you were to grant me A then I could get B from $A \supset B$ which you granted at the start.

Skeptic (3): But why should I grant you A ?

⁴ That is, the rule $\frac{}{A \succ A}$

Prover (4): Because you granted it to me at the start!

This informal dialogue is formalized as the following derivation:

$$\frac{\frac{\frac{A \supset B \succ A \supset B}{A, A \supset B \succ B} \text{Id} \quad \frac{A \succ A}{\supset E} \text{Id}}{\frac{A, A \supset B, B \supset C \succ C}{A \supset B, B \supset C \succ A \supset C} \supset I} \supset E \quad \frac{B \supset C \succ B \supset C}{\supset E} \text{Id}}{\supset E}$$

The authors provide the following explanation as how to relate the dialogue with the derivation:

The sequents offered by Skeptic are ones which entail the challenged sequent because the challenged sequent can be derived from it using only our introduction rule ($\supset I$); by contrast, the sequents offered by Prover are ones which entail the challenged sequent either because (a) the sequent offered by Skeptic can be derived from them using only elimination rules ($\supset E$) and ($\perp E$) and the structural rule (KL), or (b) they can be used to derive a sequent $\Gamma \succ \perp$ using only elimination rules or our structural rule for which there is some formula C such that from $\Gamma \succ C$ the challenged sequent can be derived using only introduction rules. (Dutilh Novaes and French 2018, p. 137)

Therefore, the natural deduction derivation conveys information about the strategy for Prover. Still, it does not represent an actual play; at most, it is a representation of the wisest possible play for the Prover. To see why, let us consider again the informal presentation of the dialogue. As the Prover claims that $A \supset B, B \supset C \succ A \supset C$, the Skeptic challenges it with $A, A \supset B, B \supset C \succ C$. Why did (s)he decide to do that? The choice is completely mysterious, unless we grant that they have the rules of the calculus in sight. In other words, they are playing *to produce* a valid proof-theoretic derivation.

I grant that this gives the proposal a wide scope and makes it easy to relate the dialogical framework with the proof-theoretic one, highlighting their similarities and confluences. But it has the severe drawback that their understanding of what is for a particular dialogue to exhibit paradoxical features (as in the Chadwick example) is very imprecise. One does not *see* what happens when Prover makes a claim such as the liar paradox, or what the difference is between a paradox being claimed by the Prover or by the Skeptic. This makes rather obscure everything that they can say about paradoxes and their proposed solutions, to which we will now turn.

Dutilh Novaes and French are interested in the substructural solutions to the semantic paradoxes. Say that \vdash is a consequence relation over a certain language, in the sense that $\Gamma \vdash A$ states that the argument with premises Γ and conclusion A is valid. We say that \vdash is *structural* if and only if the following conditions hold:⁵

Reflexivity $\{A\} \vdash A$

Monotonicity If $\Gamma \vdash A$ then $\Gamma \cup \Delta \vdash A$

Transitivity If $\Gamma \vdash A$ and $\Delta \cup \{A\} \vdash B$ then $\Gamma \cup \Delta \vdash B$

Any logic whose consequence relation fails to meet some of these conditions is called *substructural*. As the ongoing literature on semantic paradoxes has shown, there are non-trivial truth theories than can be defined in some substructural logics with nice classical recapture results (i.e., everything besides paradoxical sentences behaves as is expected in a classical theory).⁶ But “whatever restrictions on structural rules we may want to enforce, it is highly desirable that such restrictions be accompanied by independent motivation, not directly related to paradoxes” (Dutilh Novaes and French 2018, p. 130). The purpose of their paper is to provide such an independent motivation, based on the dialogical framework described above.

The authors consider four possible structural constraints: non-reflexivity, non-transitivity and two more that are of no interest here. After evaluating the relevant structural rules as games between Prover and Skeptic, they conclude that “nonreflexive solutions are to be preferred over the others” (Dutilh Novaes and French 2018, p. 147). The reason is that they found Reflexivity already suspicious:

[O]nce we move to an explicitly multi-agent context such as the Prover-Skeptic dialogues we described above, Reflexivity acquires a different meaning, as it amounts to Prover putting forward A for Skeptic to grant twice. [...] To “force” Skeptic to grant A once he has granted A is a rather pointless dialogical move, indeed a violation of pragmatic conversational norms against redundant moves [...]. In an adversarial context, such a move would most likely make Prover look silly rather

⁵ To be precise, these are the conditions that most commonly define a *tarskian* consequence relation. The term “structural” was introduced by Gentzen as a distinctive feature of certain rules of sequent calculi. The distinction is unimportant in this context though, for Tarski’s conditions and Gentzen’s rules are, in a relevant sense, equivalent.

⁶ An incomplete list of references about this topic is in Cobreros et al. 2012, 2013, Ripley 2015, Tennant 2015, Barrio, Pailos and Toranzo Calderón 2021

than triumphant; in a cooperative context, such a move would be utterly uninformative and superfluous (again, a breach of the Gricean maxim of quantity). (Dutilh Novaes and French 2018, p. 146)

So, once they turn to the consideration of the proof system with the paradoxicality operator, they argue for a constraint in the use of the (Id) rule:

As is clear from the arguments above, we think that what has gone wrong here is the reliance on structural reflexivity. Indeed, the justification given for why Skeptic cannot challenge (*Id*) earlier—namely that instance of (*Id*) must involve only formulas which Skeptic has already granted—no longer holds in this setting. This suggests that we should simply be more explicit about which formulas have been granted by the participants beforehand, instead of having this information be carried implicitly by which formulas appear in the antecedent of Skeptic offers. On this revised account, Prover and Skeptic must at least implicitly settle on a collection of sentences which constitute conversational bedrock—sentences for which no further challenge is relevant. On this picture the validity of principles like (*Id*) is not a matter of *logic* per-se, but rather one of negotiated agreement. (Dutilh Novaes and French 2018, p. 149)

I have two problems with this argument. For a start, as we have seen, the instances of (Id) are the sole condition of victory for the Prover in this setting. As quoted above, “a dialogue is won by Prover whenever the only sequents which Skeptic has available to challenge are instances of (Id)”. If this is the case, and Prover is supposed to find a way to settle these instances, it is a bit odd that later on the authors recognize that this “is a rather pointless dialogical move”, especially if logical validity is supposed to be conveyed by winning strategies for Prover. My second concern is with the final conclusion of the preceding quote: “On this picture the validity of principles like (*Id*) is not a matter of *logic* per-se, but rather one of negotiated agreement”. If we must seek a previous negotiated agreement on what instances of (Id) in which we should (or should not) rely on (or, equivalently, which are the propositions we both agree), then the subsequent dialogue (and therefore, its logic) is rather pointless. It seems that the only way to block this odd conclusion would be to expect this agreement to stem from the dialogue itself, i.e., that the interaction between the players settles which propositions are grounded and which are not. This is precisely what our account will accomplish.

3. *The Present Proposal*

I do not regard any proposal, including the one to be advanced here, as the definitive interpretation of the dialogical use of “true”, or the definitive solution to the semantic paradoxes. On the contrary, I see many open questions regarding the philosophical justification of the proposal, and its metalogic is still severely underdeveloped. I do hope that the dialogical theories given here have two virtues: first, that they provide some good ideas and interesting insights to push forward some further developments; second, that the formal expression of these ideas captures important intuitions at the level of the plays and not only of the strategies.

As Dutilh Novaes and French, I am interested in the substructural solutions to the paradoxes. But unlike them, I shall work in a full-fledged dialogical framework and not a proof-theoretic presentation interpreted in a dialogical way. Substructurality then will emerge as a consequence of reasonable decisions at the player level.

I shall begin by exposing the kind of Dialogics we will be working with. Then I shall turn to the truth theory I propose and the way in which it handles the paradoxes. The framework to be briefly exposed in the first part follows the line of Lorenzen and Lorenz (Lorenzen and Lorenz 1978, Lorenz 2001, 2010) as continued by Rahman and his colleagues and students. For a more detailed exposition, see Rahman et al. (2018, ch. 4–5) and Clerbout and McConaughey (2022).

3.1. Dialogics

Formal dialogues in this setting are games to be played by Opponent (a she) and Proponent (a he). Their identity is given by their role in the game: She has to defend the concessions and challenge the thesis while He has to defend the thesis and challenge the concessions. The distinction between *concessions* and *thesis* mirrors the distinction between *premises* and *conclusion* in a proof-theoretic presentation. The game with concessions Γ and thesis A is represented as $\mathcal{D}(\Gamma, A)$.

The meaning of the logical constants is determined by rules governing their challenges and defenses. These rules appear in tables 1, 2, and 3 (where \mathbf{X} and \mathbf{Y} are *different* players). Neutrality of topic is reached at the level of atomic formulas through a formal convention: whatever the matter they are discussing, the Proponent may claim an atomic proposition only if the Opponent has already claimed it. This is called a *Copy-Cat move*.

A *logic* is captured by a set of structural rules that determine how games are played. We are interested in the rules for classical logic:

	Conjunction		Disjunction	
Move	X claims $A \wedge B$		X claims $A \vee B$	
Challenge	Y requests \wedge_L	Y requests \wedge_R	Y requests $\vee?$	
Defense	X claims A	X claims B	X claims A	X claims B

Table 1. Local Rules (I): Conjunctions and Disjunctions.

	Negation	Conditional
Move	X claims $\neg A$	X claims $A \supset B$
Challenge	Y claims A	Y claims A
Defense	-	X claims B

Table 2. Local Rules (II): Negations and Conditionals.

	Universal Q.	Existential Q.
Move	X claims $\forall xA$	X claims $\exists xA$
Challenge	Y requests $A[a/x]$	Y requests x
Defense	X claims $A[a/x]$	X claims $A[a/x]$

Table 3. Local Rules (III): Quantifiers.

	Atomic propositions	
Move	P claims A	O claims A
Challenge	O <i>ut?</i> A	-
Defense	P <i>sic</i> (i) where i is a previous claim of A by O	

Table 4. Copy-Cat rules. **O** and **P** are the Opponent and the Proponent resp.

SR0: Starting rule A dialogue starts with the Opponent stating initial concessions (if any) and then the Proponent stating the thesis. Then each of them will choose a positive integer. This number is their *repetition rank*.

SR1: Development rule Players move alternatively. Each move is either a challenge or a defense over the moves of the other, in accordance with the Local Rules. A player can repeat a move up to as many times as the number (s)he chose as her/his repetition rank.

SR2: Copy-Cat rule Challenges and defenses of an atomic formula are governed by the rules in table 4.

SR3: Winning rule After all possible moves have been made, whoever has made the last move wins the game.

As an example, consider the dialogue in table 5. Opponent and Proponent are **O** and **P** and m and n are their repetition ranks. The outer columns keep track of the order in which moves are made. Requests are abbreviated with a question mark. The inner columns indicate which move is being challenged. A new challenge appears in a separate line, and every defense is placed alongside with its corresponding challenge. The mark “++” signals the final move.

	O			P	
				$(A \vee B) \supset B$	0
1	$m := 1$			$n := 2$	2
3	$A \vee B$	0		B	4
5	$ut? B$	4		$sic(7)$	8++
7	B		3	$\vee?$	6

Table 5. A dialogue for the thesis $(A \vee B) \supset B$. The Proponent wins.

This is just one possible outcome of the game for the thesis $(A \vee B) \supset B$. It is a game where the Proponent wins, but only because the Opponent does not play wisely. When the Proponent requests for a disjunct (move 6), she is free to choose either A or B and chooses as a defense the one that the Proponent needs to win. On a second thought, she may have realized that A was her best option: as he is constrained by the Copy-Cat rule, the Proponent is left out of options and loses the game, as shown in table 6.

	O			P	
				$(A \vee B) \supset B$	0
1	$m := 1$			$n := 2$	2
3	$A \vee B$	0		B	4
5	$ut? B$	4			
7++	A		3	$\vee?$	6

Table 6. A dialogue for the thesis $(A \vee B) \supset B$. The Opponent wins.

In this case we say that the Opponent has a *winning strategy* for this game: she has a way to counter all possible moves by the Proponent and retain victory.

Let us take a moment to meditate on the meaning of the rule SR2. If we compare the Local Rules (tables 1, 2, and 3) with the Copy-Cat rule (table 4), we see that only in the latter are the players distinguished. Without this asymmetry, the role of the players would be indistinguishable: they both would be *proponents* of their own claims. What the rule SR2 achieves is to capture within the formalism the Principle of Equality to which the Proponent should be bound: *to defend his claim on support of the concessions made by the other*.

Principle of Equality: “My reasons for stating this proposition you are now challenging are exactly the same as the ones you brought forward when you yourself stated that very same proposition” (Rahman et al. 2018, p. 8).

The games of Dialogic that incorporate the SR2 rule are called *formal* because it does not matter what is the subject at hand. In games without the SR2 rule, the players have to “do something else” to decide an atomic proposition: to perform a task or look up for a model. These are called *material* games.

Finally, here are some known facts about dialogics which will come in handy for what follows:

Fact 1. *The Proponent has a winning strategy in the game $\mathcal{D}(\Gamma, A)$ if and only if $\Gamma \vdash A$ is classically valid.*

Fact 2. *The Opponent has a winning strategy if and only if the Proponent has not.*

Proof. Clerbout 2014a, th. 1.2.1. □

Fact 3. *If the Proponent has a winning strategy in a game, then he has a winning strategy for that game with repetition rank $n := 2$ regardless of the choice of his contender.*

Proof. Clerbout 2014a, th. 1.2.2. See also Clerbout 2014b, section 4. □

Fact 4. *If the Opponent has a winning strategy in a game, then she has a winning strategy for that game with repetition rank $m := 1$ regardless of the choice of her contender.*

Proof. Clerbout 2014a, th. 1.2.2. See also Clerbout 2014b, section 4. □

3.2. Dialogical Truth Theories

We wish to capture an intuition of somewhat the following kind. Suppose we are explaining how to deal with the word “true”, understood as a dialogical notion like the one we described in section 1, to someone who does not yet understand it. We may say that we are able to challenge any sentence that is claimed to be (dialogically) true precisely under the circumstances when we can challenge the sentence itself. Conversely, we will be able to defend a claim of (dialogical) truth precisely under the circumstances when we can defend the sentence claimed to be true. Therefore, to defend a truth claim amounts to defend the sentence itself.

Let us see how we can give these ideas formal expression. Let L be the first-order language of the classical type we have been using, with a finite (or even denumerable) list of primitive predicates. Let us also assume that the language L is rich enough so that the syntax of L (say, via arithmetization) can be expressed in L , and that some coding scheme codes sentences as object constants. If A is a sentence, call $\ulcorner A \urcorner$ its code.

Suppose we extend L to a language \mathcal{L} by adding a monadic predicate $\text{TR}(x)$. The interpretation of $\text{TR}(x)$ is given by the rules in table 7.⁷

Move	X claims $\text{TR}(x)$
Challenge	Y requests x
Defense	X claims x

Table 7. Rules for the truth predicate.

It should come as little surprise that $\text{TR}(x)$ behaves somewhat like a classical double negation or a necessity operator, in the sense

⁷ Given that x is a term and not a proposition, the correct notation for the challenge and the defense in this table should be:

$$\mathbf{Y} \text{ requests/claims } \ulcorner x \urcorner^{-1}$$

This was the first notation I used, but, as an anonymous referee observed, it complicates the reading beyond necessity. So I preferred the suggested *abus de langage* (thanks).

that the defense only eliminates the prefix operator.⁸ The following theorems support this claim.

Theorem 1 (Tarski’s scheme). *The Proponent has a winning strategy for the game*

$$\mathcal{D}(\emptyset, (A \supset \text{TR}(\ulcorner A \urcorner)) \wedge (\text{TR}(\ulcorner A \urcorner) \supset A))$$

Proof. The following plays illustrate the winning strategy.

P_1 :

	O				P		
					$(A \supset \text{TR}(\ulcorner A \urcorner)) \wedge (\text{TR}(\ulcorner A \urcorner) \supset A)$		0
1	$m := 1$				$n := 2$		2
3	\wedge_L	0			$A \supset \text{TR}(\ulcorner A \urcorner)$		4
5	A	4			$\text{TR}(\ulcorner A \urcorner)$		6
7	$? A$	6			A		8
9	$ut? A$	8			$sic(5)$		10++

P_2 :

	O				P		
					$(A \supset \text{TR}(\ulcorner A \urcorner)) \wedge (\text{TR}(\ulcorner A \urcorner) \supset A)$		0
1	$m := 1$				$n := 2$		2
3	\wedge_R	0			$\text{TR}(\ulcorner A \urcorner) \supset A$		4
5	$\text{TR}(\ulcorner A \urcorner)$	4			A		8
7	A		5		$? A$		6
9	$ut? A$	8			$sic(7)$		10++

□

Theorem 2 (Tarski’s meta-scheme). *The Proponent has a winning strategy for the games $\mathcal{D}(\{A\}, \text{TR}(\ulcorner A \urcorner))$ and $\mathcal{D}(\{\text{TR}(\ulcorner A \urcorner)\}, A)$*

Proof. The following play illustrates the winning strategy for $\mathcal{D}(\{A\}, \text{TR}(\ulcorner A \urcorner))$:

P_1 :

	O				P		
0,1	A				$\text{TR}(\ulcorner A \urcorner)$		0
1	$m := 1$				$n := 2$		2
3	$? A$	0			A		4
5	$ut? A$	4			$sic(0, 1)$		6++

The following play illustrates the winning strategy for $\mathcal{D}(\{\text{TR}(\ulcorner A \urcorner)\}, A)$:

⁸ Though necessity operators do more than just that. See Clerbout 2014a; Krabbe 2006 for more on modal dialogics. Thanks to an anonymous referee for stressing the need to clarify this.

P_1 :

	O			P	
0,1	$\text{TR}(\ulcorner A \urcorner)$			A	0
1	$m := 1$			$n := 2$	2
3	$ut? A$	0		$sic(5)$	6++
5	A	4	0,1	$? A$	4

□

Theorem 3 [Right-hand Tarski’s meta-meta-scheme]. *The Proponent has a winning strategy for the game $\mathcal{D}(\Gamma, A)$ if and only if he has a winning strategy for the game $\mathcal{D}(\Gamma, \text{TR}(\ulcorner A \urcorner))$.*

Proof. Left-to-right: Assume that the Proponent has a winning strategy for $\mathcal{D}(\Gamma, A)$. If he can win without defending the thesis (that is, by only challenging the concessions) then he can win the game $\mathcal{D}(\Gamma, \text{TR}(\ulcorner A \urcorner))$ in the same way. If he can win by defending the thesis, then he can force the Opponent to claim whatever is needed for this defense, in terms of copy-cat moves, in turns j to k . His winning strategy for $\mathcal{D}(\Gamma, \text{TR}(\ulcorner A \urcorner))$ consists then in doing the same, and then to defend his defense of the thesis of any possible counter-challenge by appealing to the same moves j to k .

The *Right-to-left* case is analogous. If the winning strategy for the Proponent in $\mathcal{D}(\Gamma, \text{TR}(\ulcorner A \urcorner))$ consists merely on a challenge of the concessions, the same strategy applies to $\mathcal{D}(\Gamma, A)$. If the winning strategy includes the defense of $\text{TR}(\ulcorner A \urcorner)$, given the local rule for the truth predicate (table 7) it includes also the defense of A . This ensures the winning strategy for $\mathcal{D}(\Gamma, A)$. □

Theorem 4 [Left-hand Tarski’s meta-meta-scheme]. *The Proponent has a winning strategy for the game $\mathcal{D}(\Gamma \cup \{A\}, B)$ if and only if he has a winning strategy for the game $\mathcal{D}(\Gamma \cup \{\text{TR}(\ulcorner A \urcorner)\}, B)$.*

Proof. Left-to-right: Assume that the Proponent has a winning strategy for $\mathcal{D}(\Gamma \cup \{A\}, B)$. If he can win without challenging the concession A then he can win the game $\mathcal{D}(\Gamma \cup \{\text{TR}(\ulcorner A \urcorner)\}, B)$ in the same way. If he needs to challenge the concession A , then his winning strategy for $\mathcal{D}(\Gamma \cup \{\text{TR}(\ulcorner A \urcorner)\}, B)$ will consider a round of challenge-and-defense of $\text{TR}(\ulcorner A \urcorner)$. The Opponent has to claim A at the end of the round, in order to play strategically. Then he can challenge A as in the previous game and proceed with the same strategy.

The *Right-to-left* case is similar to the one in the last theorem. The reader may complete the proof as an exercise. □

But as the coding device allows for self-reference, in the presence of the truth predicate paradoxes emerge.

Definition 1. Let:

1. λ be the following sentence: $\neg\text{TR}(\ulcorner\lambda\urcorner)$
2. τ be the following sentence: $\text{TR}(\ulcorner\tau\urcorner)$
3. v_n be the following sentences: $\neg\text{TR}(\ulcorner v_{n+1}\urcorner)$ for all $(0 \leq n < \omega)$

It is now straightforward to prove the following result.

Theorem 5. *There are games with infinite rounds of moves.*

Proof. The games for λ , τ and v are infinite. Consider the play in table 8: move 8 is identical to move 0, which means that the defense of the thesis goes on and on endlessly. \square

(Note that repetition ranks here are irrelevant, because every new claim of λ is a new move.)

Theorem 5 makes fact 2 unavailable in \mathcal{L} . There will be games where none of the players has a winning strategy. But note that aside from these games things behave very nicely, as the following result shows.

Observation 1 [Classical conservativity]. *If all possible plays in a game are finite, then the Proponent has a winning strategy in that game if and only if the Opponent has not.*

O			P		
				λ	0
1	$m := 1$			$n := 2$	2
3	$\text{TR}(\ulcorner\lambda\urcorner)$	0		-	
5	λ		3	$?\lambda$	4
	-		5	$\text{TR}(\ulcorner\lambda\urcorner)$	6
7	$?\lambda$	6		λ	8 (!!)

Table 8. A wise play for λ .

Proof. The theorems 3 and 4 show that the truth predicate may be eliminated from strict finite games. Therefore, fact 2 holds in the fragment of the language where this elimination is available, because it is a fragment of L . \square

This suggests that the most intelligent way to deal with infinite plays would be to extend the notion of victory without altering this classical fragment. This can be done in at least two different ways, that I will call the *strict* and the *tolerant* approach. As we will see, this is a suggestive choice of words.

The *tolerant* game is obtained by changing the structural rule SR3 for the following:

SR3t: Tolerant winning rule The Proponent wins if and only if the Opponent does not make the last move.

If the game has only finite outcomes, it is clear that SR3t is equal to SR3. But when a game has infinite outcomes, *these become strategically useful for the Proponent*. As an illustration, consider the play in table 9.

	O			P	
				$A \vee \lambda$	0
1	$m := 1$			$n := 2$	2
3	$\vee?$	0		λ	4
	\vdots			\vdots	

Table 9. An infinite game where the Proponent wins.

In this game the Proponent claims $A \vee \lambda$, a disjunction where none of the disjuncts is a classical validity, so he won't be able to defend successfully any of them. But the rule SR3t gives him the victory in cases where the Opponent is unable to win. Therefore, it is in his interest to defend the thesis with the right disjunct. As he does, from move 4 on the game will mimic the play for λ , which we already know is infinite. As the Opponent will not retain the last word, the Proponent is entitled to the victory. This shows that inducing an infinite play is strategically advantageous for the Proponent.

The *strict* game in turn is obtained by changing the structural rule SR3 for the following:

SR3s: Strict winning rule The Opponent wins if and only if the Proponent does not make the last move.

Now the infinite games are advantageous to the Opponent. As an illustration we have the play in table 10.

	O			P	
				$(A \vee \neg A) \wedge \lambda$	0
1	$m := 1$			$n := 2$	2
3	\wedge_R	0		λ	4
	\vdots			\vdots	

Table 10. An infinite game where the Opponent wins.

In this play the Proponent claims $(A \vee \neg A) \wedge \lambda$, a conjunction where the first conjunct is a classical validity and the second induces an infinite play. Now the Opponent has to ask for this second conjunct if she seeks victory. Therefore, it is in her best interest to challenge the thesis by asking for the right conjunct. This shows that inducing an infinite play is strategically advantageous for the Opponent.

What is “tolerant” or “strict” in these games is the condition of victory for the Proponent: in the tolerant games it is enough that the Opponent does not retain the last word but in the strict games he has to do it himself. And it is a suggestive choice of words, since the tolerant and strict games are closely related to two well-known substructural solutions to the paradoxes.

Lemma 1. *The strict and the tolerant games are non-trivial.*

Proof. The Proponent does not have a winning strategy for the game $\mathcal{D}(\emptyset, P(a))$ either with the tolerant or with the strict rule. Therefore, there are invalid arguments in the logics induced by these games, which means that they are non-trivial. \square

Theorem 6. *The tolerant game is a non-transitive logic.*

Proof. The Proponent has a tolerant winning strategy over the games $\mathcal{D}(\Gamma, \lambda)$ and $\mathcal{D}(\Delta \cup \{\lambda\}, A)$ for all Γ, Δ and A . By lemma 1, there are $\Gamma \cup \Delta$ and A such that $\mathcal{D}(\Gamma \cup \Delta, A)$ is a game over which the Proponent does not have a winning strategy. Therefore, the inference relation induced by the tolerant game is nontransitive. \square

Theorem 7. *The strict game is a non-reflexive logic.*

Proof. The Opponent has a strict winning strategy over the game $\mathcal{D}(\{\lambda\}, \lambda)$, so the inference relation induced by the strict game is nonreflexive. \square

With all these elements in sight we may now turn to the philosophical harvest. From Lorenzen and Lorenz (1978) onwards, the standard understanding of propositions from a dialogical point of view had finiteness of plays as a constituting note:

For an entity to be a proposition there must exist a dialogue game associated with this entity, i.e., the proposition A , such that an individual play of the game where A occupies the initial position, i.e., a dialogue $\mathcal{D}(\emptyset, A)$ about A , reaches a final position with either win or loss after a *finite number of moves* according to definite rules: the dialogue game is defined as a *finitary* open two-person zero-sum game. (Lorenz 2001, p. 258; my emphasis.)

$\mathcal{D}(\emptyset, A)$ will be an open two-person zero-sum game irrespective of the winning rule considered, SR3t or SR3s. So every A conforms to the final part of this definition of proposition. It is the finitary condition that will fail in some cases. So a new terminology is in place. Say that a proposition is *dialogically grounded* if the definition of Lorenz (2001) applies. And if the condition of finiteness is dropped, call the proposition *dialogically ungrounded*.⁹

According to these new notions, the Liar (λ), the Honest (τ) and the Progressive Liar (ν) are all ungrounded. But there are also interesting differences between them. If we compare table 8 with tables 11 and 12, we can see that only in the defense of λ the Proponent manages to copy-cat moves from the Opponent (see moves 5 and 8 in table 8). Therefore, though the other two are infinite, only in this case the Principle of Equality is violated. This suggests yet another definition: a proposition is a *dialogical paradox* if the Proponent can defend (or challenge) the proposition, perform the Copy-Cat moves for all the atomic components of the proposition, and still be unable to win. The Honest and the Progressive Liar, though ungrounded, are not paradoxes in this sense, as expected.

We end this section with a brief comparison between Dutilh Novaes and French's dialogical approach to paradoxes and the present model. As we mentioned in the precedent section, Dutilh Novaes and French (2018) argue for a non-reflexive solution, and one of our theories of truth is of this kind. But we also have a non-transitive solution, and their opinion on these approaches is less optimistic. As

⁹ In a previous version of this paper, the distinction between groundedness and ungroundedness was introduced without the proper context. I am thankful to the anonymous referee that suggested that a consideration on the notion of proposition was needed.

	O			P	
				τ	0
1	$m := 1$			$n := 2$	2
3	$? \tau$	0		τ	4
5	$? \tau$	4		τ	6
	etc.			etc.	

Table 11. A wise play for τ .

	O			P	
				v_0	0
1	$m := 1$			$n := 2$	2
3	$\text{TR}(\ulcorner v_1 \urcorner)$	0		-	
5	v_1		3	$?v_1$	4
	-		5	$\text{TR}(\ulcorner v_2 \urcorner)$	6
7	$?v_2$	6		v_2	8
	etc.			etc.	

Table 12. A wise play for v .

a matter of fact, non-transitive solutions are evaluated by them as the least preferable ones:

[F]rom a dialogical perspective it seems very difficult to formulate acceptable reasons to restrict Transitivity. Indeed, it is hard to see how a proponent of a dialogical conception of logic, in terms of the Prover-Skeptic dialogues we have been developing, could possibly justify parting with it. None of the reasons adduced by proponents of nontransitive solutions seem to apply to these specific dialogues. (Dutilh Novaes and French 2018, p. 146)

From our point of view both approaches are perfectly reasonable. The rules SR3, SR3t and SR3s are three equally plausible precisifications of the rule:

a player wins if and only if the other player does not retain the last word

Structurality, non-transitivity and non-reflexivity are exclusively consequences of each of such possible precisifications:

Structurality if the first “player” is any of the two

Non-transitivity if the first “player” is the Proponent

Non-reflexivity if the first “player” is the Opponent

To be more precise: the choice between tolerance and strictness comes with the need to recover the truth of fact 2 in games with a truth predicate. But *which* option should be chosen depends on the context. For instance, Proponent may be a student and Opponent his professor. If the dialogue reconstructs an oral test, our strict theory of truth seems to be the most appropriate to norm their dialogue, because the victory of the Proponent should be his own merit. But if the Proponent is a public defender and the Opponent a district attorney, maybe the tolerant theory of truth is a better option, because in a trial the Proponent wins if he manages to delay the Opponent’s last word. Both are contexts in which the truth predicate is widely used, and though it is true that no professor, student or attorney *should* use paradoxical statements such as the Liar in their respective dialogues, this would be an empirical restriction and not a logical one.

4. *Conclusions*

There are few works in the literature concerning Dialogics and truth theories. To the best of my knowledge, the only straightforward paper that addresses this issue is Dutilh Novaes and French 2018. Still, their proposal is not as dialogical as it can be. In this paper I presented a full-fledged dialogic truth theory, with two solutions to the semantic paradoxes: the Strict and the Tolerant games for truth. These solutions came out to be *substructural*, as the Strict game is non-reflexive and the Tolerant is non-transitive.

It is not the purpose of the present work to make any particular recommendation among substructural solutions to paradoxes. If such a choice is context-dependant, as we suggested in the last paragraph of the preceding section, this at most inclines us towards a certain kind of dialogical pluralism (in the sense of Keiff 2007). But to defend this point is not the intention of this paper. My purpose is rather to provide a family of flexible instruments which can be explored simultaneously and whose fertility and consonance with intuition can be checked.

There are mathematical applications and purely technical problems which I have not mentioned in this sketch. For instance, there is the question of soundness and completeness between the strict and tolerant games and the logics *TS* and *ST*. All this will have to

be the aim of forthcoming work. For the time being, the current objectives of this inquiry have been fulfilled. Unlike Dutilh Novaes and French's, my proposal does tell what is the dialogical meaning of truth, obtains the paradoxes as a side-effect of other well-motivated definitions, and finds a way to block them while having substructurality as a *consequence* of it. I think that this is the major virtue of this proposal over theirs: that I do not need to “justify” substructurality, I cope with it (in any of its forms) as a not-so-high price to pay in exchange for a theory that is intuitive and non-trivial.

Disclaimer As a literary trait, the title, overall structure and some parts of the text in this paper mimic Saul Kripke's *Outline of a Theory of Truth*. The fragments of similar text are non-substantial though. There was no plagiarism intended.

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Received: April 19, 2022; accepted: February 13, 2023.