THE UNIVERSE OF DISCOURSE

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In his writings Quine has returned a number of times to the problem of the empty universe, and to the question of the status within logic of universes of discourse in general. My dissatisfaction with his treatment of this topic has prompted the following reflections. This is not to suggest that Quine's treatment is worse than those of most other writers on the subject; only that for various reasons he seems to me to present an especially interesting case. In what follows I divide my comments into two parts, corresponding to the two motives one has for introducing talk of "universes of discourse" into logic.

I

Why should logicians work with schemata? Surely the rationale for this practice is that results obtained for schemata can be transferred to meaningful statements. For example, after proving that the schema:

(1) $(\exists x)(y)Fxy$,

implies the schema:

(2) $(y)(\exists x)Fxy$,

we can be confident that, e.g., the statement:

(3) There is something that creates everything,

implies the statement:

(4) Everything is created by something or other.

The implication (1)-(2) is of interest only because of its applications, as for example to the case (3)-(4).

This process of application has much in common with the inferential step called "universal instantiation": the fact that (3) implies (4) is, so to speak, an "instance" of the fact that (1) implies (2). But there must be a certain relationship between schemata (1)-(2) and statements (3)-(4) in order that the application be justified. This relationship is often explained, accurately enough for our purposes, by saying that a certain feature of such statements—their form—is represented by the corresponding schemata. Thus (1) and (2) would be said to represent the forms of (3) and (4), respectively. Another way to put this is to say that (3) and (4) arise from (1) and (2) by an interpretation. "Interpreting" a schema, in this sense of the word, consists in providing a meaningful substitute uniformly for each schematic letter of the schema,[1] and also assigning an object as referent to each free variable of the schema. (To simplify matters I assume that free variables are the only individual terms.) In the above example, then, the interpretation of '*F* ① ②' as '① creates ②' yields (3) from (1) and (4) from (2).

If now we give proper definitions, it will be easy to show that knowledge of the logical properties of and relations between schemata does indeed provide us with information about the statements wich arise from them by uniform interpretation. Let us take validity (of schemata taken singly) as an example of a logical notion, since it is a bit simpler to deal with than implication. It may be defined

[1] The schematic letters are themselves meaningless, "dummy" terms. The substitute must of course be of the appropriate grammatical category: we must put a sentence for a sentence-letter, a monadic predicate for a monadic predicate-letter, a dyadic relative term for a dyadic schematic letter, etc.

thus: a schema is valid just in case every statement which has the form represented by the schema is true. Or, in terms of interpretation: a schema is valid just in case every uniform interpretation of it yields a true statement. Briefly, validity is truth under all interpretations. With such a definition we are directly justified in drawing a conclusion about a statement (namely, that it is true in virtue of its form) from a premiss about the logical status of a schema (namely, that it is valid), provided of course that the statement has the form of (i.e., arises by interpretation from) the schema.

Nevertheless a different definition of interpretation (together with correspondingly different definitions of validity, implication, etc.) is favored by most mathematical logicians: the model-theoretic or set-theoretic definition. "Interpretation" in the model-theoretic sense consists not in assigning *meanings* (or meaningful substitutes) to the schematic letters of a schema, but in assigning *extensions*—a truth-value for each sentence-letter, a set for each monadic predicate-letter, a set of ordered pairs for each dyadic schematic letter, etc.

This model-theoretic conception of interpretation has acquired such currency among mathematical logicians that it bids fair to displace the earlier notion (which may be dubbed "meaningful interpretation"), and the unqualified term 'interpretation' must now usually be understood in the model-theoretic sense. Still, the other usage is rather often found, especially in relatively non-technical discussions,[2] and is, moreover, justified by the authority of Quine's *Methods of Logic* (third ed., New York, 1972). In that book Quine gives such examples as 'Cassius is lean' and 'Cassius is hungry' as interpretations of the sentence-letters '*p*' and '*q*' (p. 42), and 'black' and 'swan' (i.e., 'is black' and 'is a swan') as interpretations of the monadic predicate-letters '*F*' and

[2] One example taken at random: R. L. Wilder, *Introduction to the Foundations of Mathematics* (2nd ed., New York, 1965), p. 24.

'*G*' (p. 97). It is true that subsequently he prefers the term 'substitution' to the term 'interpretation' when speaking of this notion.[3] But ultimately he takes the position that in the main body of the book he has left it ambiguous whether the interpretation of a monadic predicate-letter is a meaningful term (to be thought of as substituted for the letter) or a set (to be thought of as the set of things of which the term in question is to be accounted true; see *Methods*, p. 179).

This ambiguity is possible, according to Quine, because in determining the truth value of a statement which is obtained from a schema by "substitution" (i.e., by what I have called "meaningful interpretation") we need be concerned with the meanings of the terms only insofar as these determine extensions. So we do not need to provide an actual meaningful term for each schematic letter; a class, which is to be thought of as the extension of such a term, will do. Then we might as well call this class the "interpretation" of the schematic letter, and forget any concern with meanings or meaningful terms. This, naturally, is an attractive course for one who, like Quine, accepts set-theory while shying away from all talk of meanings. In any case, according to this argument, it does not matter whether we regard the interpretation of a monadic predicate-letter as a meaningful predicate or as a set.

Actually, however, this latter alternative—forgetting meanings altogether and trying to make do with sets only— is inconsistent with Quine's requirement that an interpretation of, e.g., a monadic schematic letter must determine what the letter is to be accounted *true of*; for in Quine's terminology it is meaningful terms, not sets or schematic letters, which can be "true of" things. (See, for example, *Methods of Logic*, Chapter 39). That is, it is impossible for the specification of a set to "fix the extension of a term" unless there

[3] As in the title of Chapter 26: "Predicates and Substitution." See also his *Philosophy of Logic* (Englewood Cliffs, 1970), Chapter 4, second section: "In terms of substitution."

is a *meaningful* term to have this set as its extension. And in any case the application of our logical results will still be to meaningful sentences; we will still be concerned with whether these are logically true, are logically false, imply each other, etc. So meanings (or meaningful terms) are an essential part of the picture, even if they are pushed somewhat into the background by sets.

But it might still be the case that, as Quine thinks, specifying a class suffices for interpreting a schematic letter, if by the former operation we are also, loosely, specifying a meaningful term—one that has that class as its extension. The point would then be that we should not care about just which term this is, provided we are given its extension. Now the claim that one need not specify which of the perhaps many such terms is the intended interpretation implies that any such term will do—that for logical purposes they are interchangeable, even though (as Quine himself has pointed out) the extension of a term does not determine its meaning (recall his well-known example 'creature with a heart' *vs.* 'creature with kidneys'). So if specifying the extension is enough to provide an interpretation, the different meanings associated with that extension must be irrelevant to logical purposes.

But this leads to the result that 'Every creature with a heart is a creature with kidneys' is true in virtue of its form, since (let us agree) the extensions of 'is a creature with a heart' and 'is a creature with kidneys' are one and the same.[4] That is to say, since the subject and predicate of this sentence have the same extension, the sentence would seem according to Quine to have the form '$(x)\ (Fx \supset Fx)$', and thus to be a logical truth. This result is unacceptable, and so we are forced to conclude that to interpret a predicate-letter we must specify a meaning. It is not true that only the extension of the interpreted term is important, since

[4] Cf. P. F. Strawson, "Propositions, Concepts, and Logical Truths", in *Logico-Linguistic Papers* (London, 1971), pp. 116-29; specifically p. 124.

terms with different meanings but the same extension must for logical purposes be regarded as different, not the same.

Perhaps in order to provide actual meaningful terms while still making use of extensions, Quine gives an alternative account of model-theoretic interpretation in *Philosophy of Logic.* Here specifying a set *A* as the extension of a hitherto uninterpreted predicate-letter '*F*' is regarded as giving the term 'ϵ *A*' as a meaningful substitute for '*F*'. This, however, leaves us with the problem that the same set can be designated in different ways, and different designations of the same set will give rise to different terms of the form 'ϵ *A*'. So it is not just the set, but the way in which it is specified which will determine what the interpretation is. Furthermore Quine's alternative account says in effect that a schema is valid if the sentences which arise from it by *a certain kind of* meaningful interpretation are all true. But of course our usual applications of logic will be to statements which make no use of the class-membership relation, and which thus could not have arisen by this kind of interpretation. It is then unclear what the motive could be for giving a *definition* of validity which is restricted in this way, rather than the unrestricted definition first proposed in terms of meaning.

In addition, it seems to me improper to introduce set-theoretical notions into the foundations of logic. Set theory is a rather dubious and speculative branch of mathematics, which especially since the discovery of Russell's Paradox has been surrounded by controversy. There are alternative formulations which are not equivalent, there are such apparently unsolvable problems as whether the axiom of choice and the continuum hypothesis are true, and there are special epistemological problems concerning the way in which we could acquire knowledge of sets. Logic is a much more fundamental subject, in that rational discussion is impossible without at least a rough agreement on logical principles, whereas reasonable men may disagree about set theory. Now the logician *must* deal with meanings or meaningful terms—the principles of logic are abstracted from arguments containing these, and are in turn intended to apply to such arguments generally. But there is no comparable reason to tie logic to sets; logic can and should be presented in a way which makes no set-theoretical assumptions. Therefore we should favor the definition of validity in terms of meaningful interpretation over that in terms of model-theoretic interpretation.

All this has been preliminary to the discussion of the main topic of this paper—the place of universes of discourse in logic. Let us approach this topic as follows. We may begin by observing that negation is a relative rather than an absolute notion. To take an appropriate example, the terms 'material' and 'non-material' are negations of each other, but (in spite of the verbal form) it makes no sense to ask which is really the positive term and which the negative: either one is negative relative to the other; neither one is "negative just in itself". Logicians take account of this situation by allowing that the statement 'The Empire State Building is non-material' can be obtained either from '*Fx*'—by assigning the building to '*x*' and the term 'non-material' to '*F*' —or from— '$\sim$*Gx*' by assigning the same to '*x*' and the term 'material' to '*G*'.

But how can this circumstance be accommodated using model-theoretic rather than (as in the above) meaningful interpretation? In the interpretation of '*Fx*' we will have to assign to '*F*' the *class* of non-material things; and in '$\sim$*Gx*' '*G*' must be assigned the *class* of material things. But both of these cannot be classes; for if they were, their union would be the (truly) universal class (i.e., the class that contains everything), and by Cantor's diagonal argument there can be no such class.[5] This shows us the need, on the "extensional" view, for our interpretation to include a spe-

[5] A truly universal class would be at least as large as its own power set; but by the diagonal argument the power set is always of greater cardinality.

cification of a set to serve as a "universe of discourse". Negation of a term will then be treated as complementation with respect to this universe, free variables will be assigned only things belonging to this universe, and quantifiers will be considered to range over only objects of this universe. We avoid paradox by scaling down our pretensions, and purporting to deal not with everything at once but just with the members of a certain limited domain.

Now, there is no doubt of the necessity for such a universe on the model-theoretic view. But this counts against that view, for in general the statements to which we want to apply logic are made absolutely, not relatively to a certain limited domain. When I say something which is really of the form '$(x)Fx$', it seems to me that the variable is completely unrestricted: when I say '*Everything* is F' I do not mean merely that everything *in a certain universe* is F (that would be expressed by '$(x) \ (x \in \mathrm{U} \supset Fx)$'. If, in ordinary language, quantifiers had to be treated as restricted to a certain limited universe, then one could not assert simply that everything changes, or that there are no meanings, or the like. And if one did say, for example, "Everything changes", his remark would be ambiguous, since the universe relative to which the quantifier is to be taken would not have been specified. (Contrast this with the situation in propositional logic, where it is quite clear that no such relativity enters into the notion of interpretation.) So it is a severe defect of the model-theoretic view that it requires that a universe be specified in order that an interpretation have been given; this would prevent the application of logic to many quite straightforward statements and arguments.

It is usually added that this universe of discourse must not be the empty set. That is, either we must define interpretation so that the set which is specified as the domain of the interpretation is required to be non-empty, or we must define validity as truth under every interpretation *of which the domain is non-empty*. But this exclusion of the empty set has not gone unchallenged. It is indeed plausible to ask: "Why does your definition allow any other set whatsoever to be a possible universe of an interpretation, but exclude the empty set? Is this not unwarranted discrimination?"

Quine, in his discussions of the empty universe,[6] bases his negative answer to this last question on "convenience", in that if the empty set were admitted on the same basis as other sets we would have to give up some logical laws which otherwise would be considered valid, e.g., that '$(x)Fx$' implies '$(\exists x)Fx$'. In itself this is a remarkably weak answer; one might hope for a more vigorous defense of quantification theory in its classical form. And Quine's further remarks in effect concede the case to those who prefer "inclusive quantification theory" (including the empty set as a possible universe of discourse) to the classical, "exclusive" theory. For he writes: "Usually the universe relative to which an argument is being carried out is already known or confidently believed not to be empty, so that the failure of a schema in the sole case of the empty universe is usually nothing against it from a practical point of view" (*Methods*, p. 38). This means that there are some schematic implications of classical quantification theory which may be applied to an actual argument only if the universe employed is not empty. But even if in a particular case we know (and not just "confidently believe") that the universe is not empty, logic does not tell us this; it has to be ascertained extra-logically, perhaps empirically. And Quine himself explicitly says that if we are in real doubt as to whether a universe is not empty we should use only such logical rules as are valid in the empty universe as well as in the non-empty ones (*Methods*, p. 99).

Those other logical rules, then, have a very queer status. Their validity depends on non-logical facts—and our right

[6] See *Methods of Logic*, pp. 98f.; "Quantification and the Empty Domain", in *Selected Logic Papers* (New York, 1966), pp. 220-23, specifically p. 220; "Meaning and Existential Inference", in *From a Logical Point of View* (Second ed., New York, 1963), pp. 160-67, specifically pp. 161f.

to use them depends on our beliefs about non-logical facts. But we cannot be satisfied with a logical procedure whose evaluations might have to be discarded if we turned out to be wrong about a question of existence, and thus we seem driven to the unwelcome conclusion that the logical rules which are valid for the empty universe as well as non-empty ones are the only rules we can trust, the only ones that are true principles of logic.

But I hope it is clear from my earlier remarks what is wrong with this line of thought. There is no reason to introduce *any* talk of universes of discourse, empty or non-empty, into the definitions of interpretation and validity. For these notions are best understood not in set-theoretic terms—that would indeed force a limited universe upon us—but in terms of truth and meaning, i.e., according to the definitions given above of "meaningful" interpretation and validity. To be sure, the model-theoretic definition of validity is, in a way, acceptable, at least to those who accept some standard form of set theory. But the point is that it has no independent standing: we will tolerate it only after it has been proved extensionally equivalent to the basic, intuitive definition—that in terms of "meaningful interpretation". This latter definition tells what we *mean* by 'validity'; the former definition, then, may be used only after it has been proved to pick out exactly the same class of "valid" formulas as does the latter. (In the same way each of the various *proof-theoretic* definitions of validity is acceptable only under the same condition, i.e., after a soundness and completeness proof has been given for it.)

Now, the model-theoretic definition of validity *can* be proved equivalent to the intuitive one, granting certain set-theoretic assumptions (see, e.g., *Methods*, Chapter 32, or *Philosophy of Logic*, Chapter 4, section 4), but only when the former excludes the empty set. So there is no need to puzzle over this exclusion: its purpose is simply to bring the model-theoretic definition of validity into line with the intuitive one. If we require non-empty universes, then '$(x)Fx \supset (\exists x)Fx$' turns out to be "model-theoretically valid"; without this requirement it would not be. But of course no matter how we meaningfully interpret 'F' this schema will come out true (i.e., its interpretation will be a true statement). Hence it really is "valid" in the basic sense, and hence the model-theoretic definition of validity which does not exclude the empty universe is unacceptable.[7]

II

I have shown above that if we want set-theoretic analogs of meaningful interpretation and validity we must exclude the empty universe. But this treatment of the problemt of the empty universe is incomplete; for there is another, more familiar motive for introducing universes of discourse into logic, which must be considered both for its own sake and because it seems to be Quine's main motive.

It may be grasped by noting that there is apparently a positive aspect to the use of interpretations within a limited universe, an aspect which is independent of the use of sets in giving interpretations. Quine explains it as follows: "In talking of interpretations of term letters, we do best to allow freedom in choosing the *universe of discourse*—the range of objects x relevant to the logical argument we are planning to carry through. Such freedom will commonly diminish by one the required number of terms . . . A valid term schema, then, is one that will come out true of *all* objects of *any* chosen universe under *all* interpretations, within that universe, of its term letters" (*Methods of Logic*, p. 97). It is this motive of suppressing a term from the schema which is to represent a sentence, and so providing a shortcut (sim-

[7] I might add that the question, sometimes asked in all seriousness by the "inclusive" logicians, "Couldn't there exist nothing rather than something?" is almost a paradigm case of a meaningless pseudo-question.

plifying our manipulations of schemata), rather than the set-theoretical requirement, which is uppermost in Quine's mind as he introduces universes for what he calls "Boolean term schemata", and later for quantificational schemata in general. In fact, as I have already pointed out, he insists that at this stage of his exposition he is not distinguishing clearly between meaningful interpretation and model-theoretic interpretation; nor need he do so, for if this term-suppressing device works with one kind of interpretation, it should work just as well with the other. Thus Quine is claiming that even meaningful interpretation might profitably be defined so that it involves not just the assigning of meanings to schematic letters and referents to singular terms but also the specification of a universe of discourse.

Let me take an example to show how this term-suppressing "universe-of-discourse device" works. We have already seen how the schematic implication (1)-(2) may be applied to the pair of actual statements (3)-(4). But on the original definition of meaningful interpretation we can see no immediate connection between the schemata (1) and (2) and the statements:

(5) There is some person who creates every person,

and:

(6) Every person is created by some person or other.

For the forms of (5) and (6) would be shown not by (1) and (2) but by:

(7) $(\exists x)\,(Gx\,\&\,(y)\,(Gy \supset Fxy))$,

and:

(8) $(y)\,(Gy \supset (\exists x)\,(Gx\,\&\,Fxy))$,

respectively; and sentences which have these forms cannot also have the forms (1) and (2). If, then, we wanted to evaluate the argument from (5) to (6) we should investigate the schemata (7) and (8) rather than the simpler (1) and (2).

But by changing our conception of interpretation slightly so as to include the specification of a "universe" or "domain" we can make the schematic implication (1)-(2) support the argument from (5) to (6). This "universe" is to be a set which has the extensions of the interpreted predicate-letters as subsets and the referents of the interpreted singular terms as members, and to which negation and quantification are implicity restricted. There are now several equivalent ways of explaining how the specification of this universe contributes to the meanings of the sentences which arise by interpreting given schemata. One of these is to say that the information I have just given in explaining the role of a "universe" is to be, in effect, directly incorporated into each such sentence. In the present example, if the universe is taken to be the class of persons, we will interpret '*F*' not as '—creates—' but as '—is a person who creates the person—', in order to limit it to the domain of persons; and instead of 'everything' and 'something' as readings for the quantifiers we will have 'every person' and 'some person'. (If the schema had had a free variable which we wanted to interpret as referring to Socrates, then we would have conjoined a clause 'Socrates is a person' to the rest of the sentence. And strictly speaking we must restrict negation just as we restrict quantification; for we want the negation of '*F*' to be restricted to persons just as '*F*' itself is, and so we must interpret the former as '—is a person who does not create the person—'.) In fact, restricting the predicates is unnecessary when the quantifiers and singular terms are already restricted; but the redundancy is harmless. In any case, by specifying the universe as the class of persons in addition to assigning '*F*' the meaning of 'creates' (whether restricted to persons or not) we do get equivalents of (5) and (6) from (1) and (2), respectively. That is to say, our interpretation of (1) *in the universe of persons* yields 'There is some *person* that creates every *person*' (i.e., (5)), where the term 'person' is brought in by the implicit limitation of the quantifiers to the domain of persons; and similarly for (2).

Nearly all logicians have used this device at one time or another for practical applications. It is especially convenient in a subject, such as arithmetic, where we are concerned with only a certain domain of objects. Fort it would be a great nuisance to have to write over and over again '(x) (x is a number $\supset$ - - - x - - -)' and '$(\exists x)$ (x is a number & - - - x - - -)' instead of simply '(x) (- - - x - - -)' and '$(\exists x)$ (- - - x - - -)'. How much more convenient to use the shorter formulas, while interpreting them relative to the universe of numbers! This term-suppressing device, then, has such a clear intuitive basis that we must regard it as acceptable; and by including the specification of a universe in our definition of interpretation we would simply make it acceptable by definition.

However, such a definition of interpretation seems to raise exactly the same problems as did the model-theoretic definition considered in part I above. Once again, we do not like being *required* to interpret '$(x)Fx$' in a limited domain rather than interpreting it as universal without limitation. Once again we do not like introducing set theoretical notions unnecessarily into the foundations of logic. Once again we are concerned about the neglect of meaning in favor of extension, and about the possibility of designating a set in more than one way. And once again the problem of the empty universe comes up: should it be excluded when we come to define validity, and if so, why? This last problem is especially serious. We may perhaps evade the first three by talking about interpretations relative to an *absolute general term* (such as '—is a person' or '—is a number'), or relative to a *property* or *attribute* (i.e., to the meaning of such a term; for example, numberhood, personhood), rather than to a set (the set of numbers, the set of persons). But the question would still arise whether this term or attribute has to apply to or be exemplified by anything. Quine's answer, judging by his remarks quoted earlier, appears to be "yes"; so that interpretation relative to the universe of persons (or relative to the term '—is a person', or relative to personhood) is all right, but interpretation relative to the universe of unicorns (or to the term '—is a unicorn', or to unicornhood) is not. Thus it may be an empirical matter whether a certain logical procedure is justified, since it is an empirical matter that there are persons but no unicorns. Or the applicability of the procedure might depend on the solution of a difficult philosophical problem, such as the problem whether there exist numbers.

But once again we will find that the problem of the empty universe can be easily solved if we return to our basic concepts of interpretation and validity (those I have dubbed "meaningful"). We should try to use these to justify the term-suppressing device, rather than allowing our appreciation of the utility of the device to lead us to modify our definitions by requiring the specification of a universe. After all, (7) and (8), not (1) and (2), give the forms of (5) and (6), respectively. If, then we use an investigation of (1) and (2) to reach a conclusion about the argument from (5) to (6), we must prove that the same conclusion will be reached in this way as would have been reached by the use of the longer formulas (7) and (8). And of course in order to do this we must state the appropriate conditions on the use of this term-suppressing device; that is, we must say in what sense, if any, the "universe of discourse" must be non-empty.

Let me begin by introducing some convenient terminology. Let us call (7)-(8) the "full schemata" for (5)-(6), and (1)-(2)—i.e., the schemata with a suppressed term—"short schemata". The rule for expanding a short schema into its full counterpart is to select a monadic predicate-letter, say 'G', and write '$(\exists x)\ (Gx\ \&\ \text{- - -}\ x\ \text{- - -})$', in place of '$(\exists x)$ $(\text{- - -}\ x\ \text{- - -})$' wherever the latter occurs in the short version, write '$(x)\ (Gx \supset \text{- - -}\ x\ \text{- - -})$' similarly in place of '(x)

(- - - x - - -)', add on a conjunctive clause 'Gx' if 'x' is a free variable, and do likewise for variables other than 'x'. Our aim, then, is to show that at least in a large class of cases implication holds between short schemata only if it holds between the corresponding full schemata, and furthermore we want to delimit this class of cases precisely.

In this endeavor it is natural to try to use the idea, which is often given as a heuristic explanation of the universe-of-discourse device, that for the purposes of the application we are feigning that the (actual) universe contains only persons (or only numbers, or whatever). Incidentally, this does nothing to support any sort of non-emptiness requirement, since if we can feign that everything is a person we can presumably feign that everything is a unicorn. In any case the best way to use this idea appears to be as follows. We note that if besides (7) we had an additional, tacit premiss '$(x)Gx$', we could infer (1). We already know that (1) implies (2), and, using '$(x)Gx$' again, from (2) we can get (8). Thus we can get from (7) to (8) by way of the step from (1) to (2); and the reasoning in this case can obviously be generalized.

Does this prove that it is legitimate to work with short forms rather than with the corresponding full forms? No, for there is a hitch in the reasoning: we got from (7) to (8) *via* the step from (1) to (2) only by taking our pretended asumption that $(x)Gx.$, i.e., that everything is a person, as an additional premiss. So we used the implication of (2) by (1) to prove that (7) implies (8) not absolutely but only relatively to the assumption that everything is a person. Since we do not even believe this assumption (not to speak of the lack of a *logical* warrant for believing it), the attempted justification fails: it gives us no right to work with (1)-(2) instead of the strictly correct (7)-(8).

Let us, then, make a different attempt at a demonstration. I shall do this with the help of the proof procedure given in Quine's *Methods of Logic*, Chapter 31, which is well adapted to the purpose. This is a proof procedure for the classical predicate calculus without identity, which means that for any valid proof in the predicate calculus there is a proof with the same premisses and conclusion that follows Quine's proof procedure. I will merely sketch the procedure here, referring the reader to Quine's exposition for the details.

All (schematic) proofs are to be cast in the style of *reductio ad absurdum.* One takes as his initial lines the premisses together with the negation of the conclusion, all these in prenex form with the variables of quantification relettered if necessary to avoid conflict with any free variables, and then one attempts to derive a contradiction as follows. "Begin with a great wave of EI: instantiate each existential line once, always using a new instantial variable . . . Then follow with a great wave of UI: instantiate each universal line, whether old or emergent, with each variable that is already free in the proof . . . Then another wave of EI, instantiating any further existential lines brought by the wave of UI" (*Methods*, p. 174). Continue with successive waves of UI and EI until the process terminates, either because no more instances can be obtained in this way or because a truth-functional contradiction is obtained.

For illustration, let us derive (2) from (1) by this method. First we write (1), which is already in prenex form, and then a prenex form of the negation of (2), '$(\exists y)(x) \sim Fxy$'. The first wave, of EI, produces the lines '$(y)Fzy$', and '$(x) \sim Fxz'$'; the second wave, of UI, produces 'Fzz', 'Fzz'', '$\sim Fzz'$', and '$\sim Fz'z'$'. Here we have a truth-functional contradiction between 'Fzz'' and '$\sim Fzz'$', and in any case we can get no more instances in the prescribed manner, and so the derivation is terminated. The entire derivation can be set out like this:

$$\left.\begin{array}{l}(\exists x)\,(y)Fxy\\(\exists y)\,(x)\sim Fxy\end{array}\right\}\quad \text{Initial lines}$$

$$\left.\begin{array}{l}(y)Fzy \\ (x) \sim Fxz'\end{array}\right\} \text{Wave 1}$$

$$\left.\begin{array}{l}Fzz \\ Fzz' \\ \sim Fzz' \\ \sim Fz'z'\end{array}\right\} \text{Wave 2}$$

Now this derivation can be used as a basis for a derivation of (8) from (7), these being the full forms of (2) and (1), respectively, with the quantifiers restricted to the class of G's. The construction of the parallel derivation will be especially easy if we allow ourselves two rules of inference beyond those used by Quine, namely conjunctive simplification (that '$p \,\&\, q$', implies 'q') and what I will call "conjunctive *modus ponens*" (that '$p \,\&\, q$', and '$p \supset r$' together imply 'r'). The addition of these rules, of course, will not affect the soundness of our proofs. Let us proceed, then, as follows: write the new initial lines, which are just the old ones with the quantification restricted: '$(\exists x)\ (Gx \,\&\, (y)(Gy \supset Fxy))$' and '$(\exists y)\ (Gy \,\&\, (x)(Gx \supset \sim Fxy))$' (these are (7) and a form of the negation of (8), respectively). Proceed with a wave of EI as in Quine's Wave 1, which we shall also call Wave 1, obtaining: '$Gz \,\&\, (y)(Gy \supset Fzy)$' and '$Gz' \,\&\, (x)(Gx \supset \sim Fxz')$', But this time we insert another wave ("Wave 1½"), whose lines are obtained from the lines of Wave 1 by conjunctive simplification, that is, by dropping the first half of each of the conjunctions just obtained and leaving the second half. In the example this will result in the lines: '$(y)\ (Gy \supset Fzy)$' and '$(x)\ (Gx \supset \sim Fxz')$'. Then proceed with Wave 2 (of UI) as in Quine's proof procedure, obtaining from the lines of Wave 1½: '$Gz \supset Fzz$', '$Gz' \supset Fzz'$', '$Gz \supset \sim Fzz'$', and '$Gz' \supset \sim Fz'z'$', respectively. Then again after this wave is completed we add another ("Wave 2½"), each of whose lines is derived from a line of Wave 2 (together with a previous line) by conjunctive *modus ponens*; in effect this drops the antecedent of each conditional obtained in Wave 2. In the example this gives us: 'Fzz', 'Fzz'', '$\sim Fzz'$', and '$\sim Fz'z'$'. Here, just as in the original derivation, we have a truth-functional contradiction between 'Fzz'' and '$\sim Fzz'$'. The whole derivation, then is as follows:

$$
\left.\begin{array}{l}
(\exists x)\ (Gx\ \&\ (y)\ (Gy \supset Fxy)) \\
(\exists y)\ (Gy\ \&\ (x)\ (Gx \supset \sim Fxy))
\end{array}\right\}\ \text{Initial lines}
$$

$$
\left.\begin{array}{l}
Gz\ \&\ (y)\ (Gy \supset Fzy) \\
Gz'\ \&\ (x)\ (Gx \supset \sim Fxz')
\end{array}\right\}\ \text{Wave } 1\tfrac{1}{2}
$$

$$
\left.\begin{array}{l}
(y)\ (Gy \supset Fzy) \\
(x)\ (Gx \supset \sim Fxz')
\end{array}\right\}\ \text{Wave } 2\tfrac{1}{2}
$$

$$
\left.\begin{array}{l}
Gz \supset Fzz \\
Gz' \supset Fzz' \\
Gz \supset \sim Fzz' \\
Gz' \supset \sim Fz'z'
\end{array}\right\}\ \text{Wave } 2
$$

$$
\left.\begin{array}{l}
Fzz \\
Fzz' \\
\sim Fzz' \\
\sim Fz'z'
\end{array}\right\}\ \text{Wave } 2\tfrac{1}{2}
$$

This example illustrates a general method of taking a proof by Quine's procedure involving short schemata and generating from it a proof in the modified style involving the corresponding full schemata. I trust that the example sufficiently justifies the method that it will not be necessary to produce a tedious proof, by induction on the number of waves in the original derivation, of the claim that such a parallel derivation is generally obtainable. There are, however, a couple of special features of the example which deserve notice. If a *premiss* of the original (short-form) argument contained one or more free variables, then the corresponding premiss of the parallel (full-form) argument would contain the same number of conjunctive clauses of the form 'Gz' ('Gw', etc.). In the modified Quinean proof for such an argument these would first have to be dropped by conjunctive simplification, and then one would proceed as in the illustration. Also, if one of these variables ('*z*', let us say) was free also in the original *conclusion*, the clause '*Gz*' would in effect appear as the antecedent of a conditional in the corresponding initial line of the full-form derivation, and would have to be dropped by conjunctive *modus ponens*. But if the original *conclusion* contained a free variable ('*z*') which was free in no *premiss*, then the parallel full-form derivation would not go through; nor would it if the initial lines, while devoid of free variables, were none of them existential quantifications.

The reasons for failure in these two cases are similar. In Quine's proof procedure any free variables of the initial lines are available as instantial variables for the second wave, and for all subsequent waves of UI; and if there are niether free variables nor existential initial lines a variable is arbitrarily selected for use in the second wave, the first wave being considered "empty" (see the example, *Methods*, p. 174). In the parallel full-form derivation the second wave will yield conditionals, to which we want to apply conjunctive *modus ponens* in the second-plus-a-half wave. But the antecedent of such a contidional will be available in a previous line only when the instantial variable was introduced by EI or was free in a premiss. If the variable '*z*' was free only in the negation of the conclusion then, while the full form of the conclusion itself would imply '*Gz*', the negation of this would not. And of course if the variable '*z*' was introduced for lack of both free variables and existential initial lines, we would not have '*Gz*' available to us. The simplest illustrations of the two cases of failure are the short argument-forms:

$$\frac{(x)Fx}{Fz,}$$

and:

$$\frac{(x)Fx}{(\exists x)Fx,}$$

which are valid even though their full counterparts:

$$\frac{(x)\ (Gx \supset Fx)}{Gz \,\&\, Fz,}$$

and:

$$\frac{(x)\ (Gx \supset Fx)}{(\exists x)\ (Gx \,\&\, Fx),}$$

respectively, are not. But except for these two kinds of cases we can always find a parallel proof in full forms for each proof in short forms.

What has been shown, then, is that whenever certain schemata (the premisses) imply a schema (the conclusion), and there are no free variables in the conclusion which are not free in a premiss, and either there is an existential or singular premiss or there is a universal conclusion, then the long or full forms of the premisses—obtained by substituting a schema of the form '$(\exists x)\ (Gx \,\&\, \cdots x \cdots)$' for each subschema of the form '$(\exists x)\ (\cdots x \cdots)$', a schema of the form '$(x)\ (Gx \supset \cdots x \cdots)$' for each subschema of the form '$(x)\ (\cdots x \cdots)$', and a schema of the form '$Gz \,\&\, \cdots x \cdots$' for each complete schema of the form '$\cdots z \cdots$' imply the full form (similarly obtained) of the conclusion. This justifies us in suppressing the term 'G' in a wide variety of cases, and using the short forms of schemata when working on a problem of implication rather than the more complicated full forms. (Of course the converse is obvious: if there is implication between the full forms there is also implication between the short forms, as we can see by substituting '$Jx \vee \sim Jx$' for 'Gx'.) So we have achieved by this demonstration the purpose for which limited universes of discourse are most often introduced into the definition of interpretation while avoiding the confusion—especially about the empty set—which usually accompanies that introduction.

By considering the remaining cases—those for which suppression of 'G' is not legitimate—in the light of the foregoing argument, we can better understand the true significance both of the requirement of non-emptiness for universes of discourse and of the requirement that interpreted singular terms designate members of the universe of discourse. Both these requirements are seen to be confusingly stated limitations on the applicability of the device of suppressing a term. The real points are that the full forms of the premisses of an argument must imply '$(\exists x)Gx$' if the conclusion is existential, and that they must imply 'Gz_1', 'Gz_2', etc. for each variable 'z_1', 'z_2', etc. which is free in the conclusión. It is required not that 'z_1', 'z_2', etc. be interpreted as designating things that really exemplify the property that 'G' is interpreted as meaning, nor that there actually be things that have this property, but only that the (interpreted) premisses imply these conditions. Of course, if they do not, the full-form argument will not be valid, even if the short-form one is so. Such short-form arguments are sometimes said to be invalid for the empty universe; this simply means that they are valid but their full-form counterparts are not.

Another way of looking at the matter is this. What is called "interpreting an argument-schema in a limited universe of discourse" really amounts to treating the schema as a stand-in for what we might call its corresponding "augmented full form". The latter is obtained by replacing the premisses and conclusion by their corresponding full forms (this is what is involved in interpreting each of them separately in the limited universe), and adding premisses 'Gz_1', 'Gz_2', etc. for each variable 'z_1', 'z_2', etc. which is free in the conclusion but not in a premiss, and adding also the premiss '$(\exists x)Gx$' if the conclusion is existential while none of the premisses are either existential or singular. Between the original argument-schema and its augmented full form there is perfect correspondence—the one is valid if and only if the other is.

Now suppose we wish to show that a given argument-schema is invalid by providing an interpretation which makes the premisses come out true and the conclusion false. It may be most convenient to provide this interpretation "in a limited universe". This means that implicitly we are working with the augmented full counterpart of the given argument-schema. The requirement that the universe not be empty and the requirement that all singular terms be interpreted as designating members of the universe then serve as reminders that our interpretation must not only make the full forms of the original premisses true and the full form of the conclusion false but must also make any additional premisses of the form 'Gz' or '$(\exists x)Gx$' that are contained in the augmented full argument-schema come out true. The additional premisses might tend to be forgotten because they have no counterparts in the short form of the argument-schema.

Now if we intend to supply such an interpretation in the universe of, say, numbers, and later we discover that there really are no numbers, then we have discovered that our interpretation does not do the job for which we intended it. It is not any the less proper an interpretation for that; it merely fails to be an interpretation that *makes* all the premisses of the augmented full argument-schema come out true and the conclusion false. Thus no logical laws are threatened by the possibility that there are no numbers. For even if we discovered that in fact there were none, no arguments that we had thought were valid would have turned out to be invalid: we would merely have discovered that a premiss ("There exist numbers") which we thought was true is really false.

I conclude that the device of suppressing a term and thus obtaining simpler schemata for manipulation can be understood quite well without reference to so-called "universes of discourse". The most that can be said for *defining* interpretation or validity with reference to non-empty universes in order to allow such term-suppression is that it will not lead us into any logical mistake; but it has precisely the advantages of theft over honest toil, in that what is essentially a shortcut method is pronounced sound by definition. And at worst it engenders confusion about the status of the non-emptiness requirement. If instead of altering our definitions we try to *prove* that suppressing a term is sometimes acceptable procedure, we quickly see that it is so only if the existential quantification of the term suppressed is implied by the premisses of the relevant argument-schema together with the negation of the conclusion. *This* is all the "non-emptiness" we need or want. And if the reasons given here for banishing talk of universes from the foundations of logic are cogent, we need not take seriously the suggestion that certain logical principles should be rejected in the interest of "interpretations in the empty universe of discourse".

RESUMEN

El presente es un artículo en el que se discute, de manera crítica, el status, dentro de la lógica, del universo vacío en particular y de los universos de discurso en general. La discusión se centra alrededor de la posición de Quine a este respecto.

I

De manera preliminar se enfatiza la utilidad que tienen las fórmulas-esquemas en lógica en tanto que las mismas pueden convertirse en enunciados significativos. Las fórmulas representan la forma de los enunciados que de ellas pueden obtenerse mediante una interpretación que se caracteriza como la asignación uniforme de un sustituto significativo a cada letra esquemática de la fórmula-esquema así como un objeto como referencia a cada una de sus variables libres.

Se define la validez de una fórmula-esquema de la siguiente manera: una fórmula-esquema es válida sólo en caso de que sea verdadero todo enunciado que tenga la forma representada por el esquema. O bien, esto mismo, en términos de interpretación: una fórmula-esquema es válida, sólo en caso de que toda interpretación uniforme de la misma, tenga como resultado un enunciado verdadero. Esta caracterización de interpretación y la definición de validez se contrastan con las que se ofrecen en términos de conjuntos, en donde una interpretación no consiste en asignar, a las letras esquemáticas del esquema, *significados* sino *extensiones* (un valor de verdad a cada letra oracional, un conjunto a cada letra de predicado monádico, un conjunto de pares ordenados a cada letra esquemática diádica, etc.).

Ahora bien, si se acepta la versión extensional y se insiste en mantener que la interpretación de una letra esquemática es algún conjunto, los diferentes significados asociados a esa extensión deben ser irrelevantes para propósitos lógicos. Pero entonces se tendría el resultado inaceptable de que, suponiendo que "Creatura con corazón" y "Creatura con riñones" tienen la misma extensión, "toda creatura con corazón es creatura con riñones" tiene la forma "$(x)\ (Fx \supset Fx)$". Con esto se concluye que una interpretación se determina no por la extensión sino por la forma como ésta se especifica.

Otro inconveniente de una interpretación extensional conjuntista es que la lógica es un estudio mucho más básico que la teoría de los conjuntos. De aquí que una definición de validez en términos de interpretación significativa ha de preferirse a una interpretación en términos de conjuntos.

Dejando atrás estos preliminares, se señala que en una interpretación extensional conjuntista el universo de discurso ha de precisarse de antemano para evitar que una clase o su complemento no sean conjuntos; el complemento de un conjunto, en este caso, será siempre relativo al (subconjunto del) conjunto que se asigne como universo de discurso. Pero esta relatividad (limitación) no se tiene en múltiples enunciados del lenguaje ordinario a los que deseamos aplicar la lógica y que se expresan de manera absoluta, no relativa. Por otra parte, también se añade la restricción de que los universos de discurso han de ser conjuntos no vacíos. Para fundar esto se ofrece como razón (Quine) que de aceptarse el conjunto vacío tendríamos que renunciar a ciertas leyes lógicas. Pero de esto se sigue, entonces, que ciertas leyes lógicas son tales, sólo por motivos extra lógicos. En cambio, esto no sucede si entendemos las nociones de interpretación y de validez en términos de verdad y significado conforme a las definiciones apuntadas en un principio.

II

Una motivación más familiar que la anterior para introducir universos de discurso en lógica es que, conforme a Quine, mediante su introducción podemos reducir en uno, el número de términos requeridos en un esquema. De esta manera se simplifican los esquemas y su manejo. Ahora bien, conforme a esta versión, la especificación de un universo de discurso sería conveniente tanto para una interpretación conjuntista como para una como la aquí propuesta en términos de significado. Pero los mismos problemas que surgían previamente acerca de la interpretación conjuntista propuesta en I, vuelven aquí a surgir: (1) hay esquemas como "$(x)Fx$" que nos gustaría interpretar como universales sin limitación, en lugar de hacerlo dentro de un universo limitado; (2) no es grato introducir nociones de teoría de los conjuntos en los fundamentos de la lógica; (3) se pretende, una vez más, excluir el significado a favor de la extensión y existe la posibilidad de designar un conjunto en más de una forma y (4) una vez más se presenta el problema del universo vacío: ¿hemos de excluirlo al definir la validez? Y, si así lo hacemos, ¿por qué? Quizás se podrían eliminar los tres primeros problemas hablando, en lugar de conjuntos, de términos y atributos, pero sería preciso preguntarse si este término o atributo ha de ser ejemplificado por algo. Quine, a esto, parece responder que sí y así parecería ser que un problema lógico (validez) ha de resolverse mediante métodos extra lógicos (nuestro conocimiento de si el universo es o no vacío).

El problema se presentaría, p.e. en el caso de la fórmula-esquema "$(x)Fx \supset (\exists x)Fx$" o del argumento esquema correspondiente, a saber

$$\frac{(x)Fx}{(\exists x)Fx}$$

el que, se dice, es válido sólo en universos no vacíos. Pero, a la conclusión a la que se llega, es que, lo anterior, sólo quiere decir que el argumento es válido pero no así su contraparte formal completa. Esto es, la contraparte formal completa de tal argumento sería

$$\frac{(x)\ (Gx \supset Fx)}{(\exists x)\ (Gx \ \&\ Fx)}$$

donde "G" es una letra de predicado monádico que especifica al conjunto en el que ha de darse la interpretación. Ahora bien, al introducir un universo de discurso, no tenemos ya que especificar el conjunto sobre el que se va a dar la interpretación, pues asumimos que todos los individuos a los que hacemos referencia son elementos de ese universo y así podemos pasar al argumento esquema considerado aquí en primer lugar.

Es claro que la invalidez del argumento esquema completo se puede eliminar añadiendo la premisa "$(\exists x)Gx$" y este procedimiento se puede generalizar de la siguiente manera: Lo que se denomina "interpretar un argumento esquema en un universo limitado de discurso" es tanto como tratar al esquema como representante de lo que podríamos llamar su correspondiente "forma aumentada completa". Esta última se obtiene remplazando las premisas y la conclusión por sus formas completas correspondientes (esto es lo que se encuentra implícito al interpretar cada una de ellas por separado en el universo limitado), y añadiendo las premisas "Gz_1", "Gz_2", etc. por cada variable "z_1", "z_2", etc. que aparezca libre en la conclusión pero no en alguna premisa, y añadiendo también la premisa "$(\exists x)Gx$" si la conclusión es existencial y ninguna de las premisas es existencial o singular. Entre el argumento esquema original y su forma aumentada completa hay una correspondencia total: uno es válido sí y sólo sí el otro lo es. A este argumento esquema completo se dará una interpretación significativa (no conjuntista), de la manera señalada en I.

Concluyo, pues, que el artificio de suprimir un término y así obtener esquemas de manipulación más simple puede entenderse muy bien sin hacer referencia a los llamados "universos de discurso".