NEGATION, DENIAL AND POSSIBILITY

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Plymouth Meeting

Abstract: In a recent article, John A. Barker has advanced various proposals about conditional statements which appear to be either false or unhelpful. The present paper advances objections to Barker's proposals in the following steps: First, two theories of impredicability —one by Fred Sommers and one by A. N. Prior— are discussed, and a later-to-be-helpful distinction used in Sommers theory between negation and denial is described. Next, it is shown that, in contradiction to Barker, it is possible to contrapose what Barker calls "counterconditionals" —i.e., denials of conditional, or "even if" statements— by one of three methods: (a) by removing ambiguity in the antecedent, (b) by removing ambiguity in the consequent, (c) by removing ambiguity in the antecedent and consequent simultaneously. Next, by similar methods, it is shown that counterconditionals are —in contradiction to Barker— subject to *modus tollens* and *modus ponens*. And finally, two apparently problematic trains of reasoning which Barker thinks are elucidated by his "conditional-thetical" distinction are shown to be adequately handled by already-existing logical methods.

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In his article "Predicability" [S], Fred Sommers has suggested that the denial of a statement P, viz., "It is not the case that P", ought to be distinguished from the negation of P, viz., the employment of the term "not" at some judicious but not precisely defined place in the interior of P. The reason for Sommers' position is that it facilitates the handling

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of statements which make illegitimate predications. For example, the statement

(1) The equator is dirty.

and its negation

(2) The equator is not dirty.

are both illegitimate predications since the equator is not the sort of thing which can be dirty or not dirty; so Sommers suggests that we regard (1) and (2) as both false, but treat the denials of these statements, viz.,

(3) It is not the case that the equator is dirty.

and

(4) It is not the case that the equator is not dirty.

as both true. This, then, will preserve the Law of the Excluded Middle by permitting us to say that either a statement or its *denial* must be true, rather than that either a statement or its *negation* must be true. Sommer's suggestion, however, assumes that we are always able to distinguish legitimate from illegitimate predications, but in fact this is not the case, or at least Sommers —on whom the *onus probandi* falls— has not shown it to be the case. For consider the following statement, which Sommers takes to be an illegitimate predication:

(5) The sky is (not) a man.

It seems apparent, at least to this writer, that such a statement might be meaningfully made by a believer in some peculiar (or perhaps ordinary) religion who believed that the sky is a gaseous god, and that this god is a man (or at least a man in some essential sense).

A theory which attempts to deal with the problem of im-

predicability in a somewhat different way is that of Prior [P], who suggests that all illegitimate predications make implicit false existential assertions which, if made explicit, can then be negated, thereby making unnecessary the distinction between negation and denial. For example, by Prior's theory, the statement

(6) The present King of France is bald.

ought to be written

(7) There exists x such that x is the present King of France and is bald.

The falsity of this latter statement may then be asserted by the negation of (7), viz.,

(7.1) There does not exist x such that x is the present King of France and is bald.

The fact that the approach of Prior is different from that of Sommers, however, is not sufficient reason to suppose that the two theories are incompatible. Indeed, we note that the inability to legitimately predicate dirtiness of the equator, which Sommers discusses, arises from an essential incompatibility between the object of predication and what is predicated; whereas the inability to predicate baldness of the present King of France occurs merely as a result of the inessential or accidental fact that there is no present King of France. If, then, we designate these two different sorts of impredicability as, respectively, *essential* and *accidental* impredicability,[1] we may say that Sommers' theory deals with the

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[1] This distinction is intended to suggest the distinction made by Aristotle between "essential" and "accidental" properties, the standar example of which is that man is said to be "essentially" rational —because no object could be a man without being rational— but only "accidentally" earth-dwelling — since a man might live on the moon and still be reckoned a man. A similar but less

former while Prior's deals with the latter, and thus that the two theories are complementary rather than incompatible.

But irrespective of how we view Sommers' and Prior's theories, the distinction which Sommers makes between negation and denial is a useful one. For consider the two statements

(8) Nuclear war will occur.

(9) Nuclear war will definitely occur.

There appears to be no essential difference between (8) and (9) except that (9) is more emphatic; but if we consider the negation, denial, and denial of the negation of (8), respectively, viz.,

(10) Nuclear war will not occur.

(11) It is not the case that nuclear war will occur.

(12) It is not the case that nuclear war will not occur.

we see that (8) and (12) are equivalent, as are (10) and (11), whereas if we consider the negation, denial, and denial of the negation of (9), respectively, viz.,

(13) Nuclear war will definitely not occur.

(14) It is not the case that nuclear war will definitely occur.

(15) It is not the case that nuclear war will definitely not occur.

We see that (14) is equivalent to

(14.1) It is possible that nuclear war will not occur.

metaphysical (and hence less controversial) distinction than Aritsotle's is that which may be made between *defining* and *supplemental* properties, i.e., between those properties which define a class by intension and those which an object in a given intensionally-defined class may have in addition to the defining properties of the class.

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while (15) is equivalent to

(15.1) It is possible that nuclear war will occur.

It is apparent, however, that (14) and (15) are equivalent in
content, altho not in emphasis (they are comparable, perhaps,
to speaking of a bottle as being partly empty as opposed to
being partly full), while in contrast, (9) and (13) are pre-
cisely opposite in meaning. Now it might be thought, par-
ticularly from observing (14.1) and (15.1), that (9), (13),
(14) and (15) are really modal statements, altho we might
hesitate to press this argument in view of the fact that we
would not normally say that (9) is equivalent to

(16) Nuclear war will necessarily occur.

unless we interpret "necessarily" in a very loose sense. It
does seem, however, that the four statements in question are
statements of *probability* —i.e., it seems most reasonable to
think of (9) as assigning nuclear war a probability of 1, of
(13) as assigning it a probability of 0, and of (14) and (15)
as assigning it some intermediate but indefinite probability—
and accordingly it would seem that we should look upon
their progenitors, viz., (8), (10), (11) and (12), as merely
ambiguous or incomplete statements.

Now an important application of the ideas which we have
so far developed is to certain problems of hypotheticals which
have recently been discussed by Barker [Ba]. To explain, we
first suppose that Tom and Dick are watching Smith play
chess, and Tom makes the following statement:

(17) If Smith should lose his rook, he will resign.

We note that (17) can be contraposed, as follows:

(18) If Smith does not resign, he will not have lost his
      rook.

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The contraposability of (17), however, does not seem to be shared by a statement made by Dick, who disagrees with Tom's assessment of the situation, and who therefore asserts a denial of (17), viz.,

(19) If Smith should lose his rook, he will not resign.[2]

for the apparent contraposition of (19) is

(20) If Smith does resign, he will not have lost his rook.

which, as Barker notes, is "absurd", and from which Barker draws the conclusion that "counterconditionals" (as he calls denials of conditionals) are not subject to contraposition. The statement (19), however, is really ambiguous in a number of different ways, so that by restating (19) in order to resolve the ambiguities, contraposition will be possible after all, in contradiction to Barker. To be specific, if we clear up an ambiguity in the antecedent by writing (19) as

(21) If Smith should lose only his rook, he will not resign.

we see that (21) may be contraposed without difficulty as

(22) If Smith does resign, he will not have lost only his rook.

There is, however, a second way of resolving the problem, which is to clear up an ambiguity in the consequent of (19) by considering whether (19) should be interpreted as

(23) If Smith should lose his rook, he will definitely not resign.

[2] Altho we give Barker's example here verbatim, it is evident on reflection that its meaning is better expressed as "*Even* if Smith should lose his rook, he will not resign", or perhaps "If Smith should *do no worse* than to lose his rook, he will not resign." (Barker, of course, avoided the former in order to emphasize the relation of (17) and (19).) The reader should be aware, therefore, that what we do in the argument which follows is to transform the "even if" statement into an ordinary "if-then" statement, which is then automatically contraposable.

The contrapositive of (23), however, is

(24) If Smith does not definitely not resign, he will not have lost his rook.

or, more clearly,

(24.1) If Smith possibly resigns, he will not have lost his rook.

but this, of course, is absurd. Accordingly, we are led to consider the possibility that the consequent of (19) should be interpreted as

(25) If Smith should lose his rook, he will not definitely resign.

This, of course, is equivalent to

(26) If Smith should lose his rook, it is possible that he will not resign.

But can (26) be contraposed? If we attempt to do so, we get

(27) If Smith's not resigning is not possible, then Smith will not lose his rook.

or, more clearly,

(27.1) If Smith's resigning is certain, then Smith will not lose his rook.

but this, again, is clearly absurd. Nevertheless, if we grant the truth of (26), it follows that

(28) If Smith should lose his rook, it is possible he will resign.

and in fact, it could be maintained that (26) and (28) are, in essence, equivalent, in the sense that (14.1) and (15.1) are equivalent. But (28) can be sensibly contraposed, viz.,

(29) If Smith's resigning is not possible, Smith will not lose his rook.

Now while it is an interesting fact that great strides can be taken in clearing up the problems of Barker's hypotheticals by resolving ambiguities in either the antecedent or the consequent, the ideal situation is evidently to resolve such ambiguities simultaneously. To do this, therefore, we seem to have exactly two alternatives for the interpretation of (19), viz.,

(30) If Smith should lose only his rook, he will definitely not resign.

(31) If Smith should lose only his rook, he will not definitely resign.

Now the contrapositive of (30) is

(32) If Smith does not definitely not resign, he will not have lost only his rook.

or, more clearly,

(32.1) If it is possible that Smith resigns, then he will not have lost only his rook.

while if we rewrite (31) as

(31.1) If Smith should lose only his rook, it is possible he will not resign.

its contrapositive is seen to be

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> (33) If it is not possible that Smith will not resign, then he will not have lost only his rook.

or, more clearly,

> (33.1) If Smith (definitely) resigns, then he will not have lost only his rook.

Both (32.1) and (33.1), however, are clearly legitimate contrapositives of their respective positives, tho both should perhaps be clarified by adding to the end of each of them the phrase "but other things as well".

Now in continuing his discussion, Barker asserts that the following, altho apparently a legitimate *modus tollens* argument, is actually illegitimate:

> (34) If Smith should lose his rook, he would not resign. But he will resign. Therefore he will not lose his rook.

If, however, (34) is reworded so as to eliminate the sorts of ambiguities we have discussed by interpreting (19) as (31), then the argument becomes

> (35) If Smith should lose only his rook, he would not definitely resign. But he will (definitely) resign. Therefore he will not lose only his rook.

and is seen to be valid. Similarly, if (19) is interpreted as (30), and if the corresponding change is made in the second sentence of the argument, then the argument becomes

> (36) If Smith should lose only his rook, he would definitely not resign. But he will not definitely not resign. Therefore he will not lose only his rook.

and is also seen to be valid, altho the argument

(37) If Smith should lose only his rook, he would de-
finitely not resign. But he will (definitely) resign.
Therefore, he will not lose only his rook.

is not valid. A parallel analysis applies to Barker's claim of
the invalidity of a *modus ponens* argument similar to (34)
(p. 338). This, however, in conjunction with our analysis of
(34) and (19), seems to refute Barker's view that counter-
conditionals are not subject to either contraposition, *modus
tollens* or *modus ponens*, providing of course that a proper
interpretation is given to such statements.

In closing, it seems appropriate to remark that, of two
further problematic arguments which Barker purports to elu-
cidate by means of what he calls the "conditional-thetical"
distinction —a distinction which he initially advanced to deal
with the problems of counterconditionals— one of these is
quite simply laid to rest by the existential mechanics of Prior
which were discussed at the beginning of this paper; and the
other is taken care of by conventional logical observation.
The first of these problematic arguments is

(38) If John has children, then not all John's childrens
are asleep. But all his children are asleep. So he has
no children.

This argument, tho patently absurd, appears at first thought
to be justifiable by *modus tollens*; but if restated with its
existential assertion made explicit, it can easily be shown to
be false. In particular, the statement in (38) "But all his
children are asleep" presupposes that the set of John's chil-
dren is nonempty; so the first two sentences of (38) should
be written as

(39) If John has children, then not all John's children

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are asleep. John has children and all John's children are asleep.

(39), of course, is a contradiction; and since some logicians opine that a contradiction implies any statement, it would follow from this that (39) implies that John has no children — but of course, by the same token, it would also imply that John *does* have children, which is evidently not the result intended to be obtained by Barker. I, however, along with von Wright [V] and a few other ornery logicians, do not accept that a contradiction implies any statement, and on this basis, I take (39) to imply only that John has children and that these children are both all asleep and not all asleep; so by this reasoning (39) does not imply that John has no children.

Now the second problematic argument which Barker mentions is the following:

(40) If not all John's children are asleep, then John has children. But John has no children. So all John's children are asleep.

Now the fallacy in this argument (which appears at first to be justifiable on the basis of contraposition) is quite plainly that it confuses the distinction between language and metalanguage, and in particular that the implication of the first sentence (that not all John's children are asleep implies that John has children) is a metalinguistic implication, whereas (40) is not, taken as a whole, metalinguistic. Elsewhere I have discussed at length [Br, ch 7] the disastrous results of such confusion, so there is no reason to go into the matter here.

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