¿DEDUCIBILITY IMPLIES REVELANCE? A NEGATIVE ANSWER (I)*
(On the philosophical status of relevant logic)

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CONTENTS:

*This paper is published in two parts, to appear in consecutive issues of *Crítica*. The first part covers the Introduction and Section I and II of the Summary detailed in the text. (To help the reader's orientation, the Summary and the Bibliography of the paper are entirely transcribed in this issue).

A preliminary version of this paper was presented in the Instituto de Investigaciones Filosóficas at UNAM, and at the Sociedad Argentina de Análisis Filosófico. The comments, especially those of the commentators (José Antonio Robles at UNAM and Gladys Palau at SADAF) were very useful for the final version, Alvaro Rodríguez helped me to solve a problem on conditionals posed in the original version; but for the sake of thematical unity I have the treatment of this and other problems on conditionals to another paper.

1. Simplification.
2. Addition.
3. Transitivity.

IV. Philosophical *status* and usefulness of relevant logic.

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INTRODUCTION

According to the definitions of *validity, logical entailment* and *deducibility* offered in the usual textbooks of logic, an inference such as 'it rains and it does not rain; therefore, the moon is made of cheese' is *valid,* its premises *logically entail* its conclusion, and this is *deduced* from those.[1] The current logical systems, built in accordance with these definitions allow for a formal justification of that inference. But the logical judgment shocks the layman's intuitions, which tend to consider "illogical" and unsound an argument like the one cited above, whose premises 'have nothing to do with the conclusion'. Many logicians have adhered to this intuition and sustain that there is no deducibility if there is no connection between the contents of premises and the conclusion; that is, if there is no "relevance" between premises and conclusion. In harmony with this idea, formal systems that do not allow reasonings like that mentioned above have been designed, and they permit the building of a deducibility relation different from the classic one. These systems are generally known as "relevant logics".

The main task of this paper is to critically analize the claim that the usual classic analysis of the notion of deducibility is, in a sense, wrong, and should be replaced by another one, along the line indicated by some relevant logic. The first two sections are devoted to the book *Entailment* by Anderson and

[1] Notice that usual technical terminology permits the logical judgement to be formulated in three equivalent ways. It could be noted also that *deducibility* is understood as the converse of *logical implication.*

Belnap[2], the most complete and elaborated work on relevant logic, because of the treatment *in extenso* of the formal and philosophical aspects of the problem. In I, I describe briefly the content of this work and some of its philosophical thesis; in II, I examine the arguments in it, offered against the classical analysis of deducibility. In section III, I consider arguments from other authors with similar views. Finally, I deal in section IV with the status and usefulness of the formal systems for relevant logic. The general thesis of this article is that the attack on the classical analysis of deducibility is ill-founded, and that the necessity of replacing it by something taken from a relevant logic is far from having been proved. In the last section it is suggested that, nevertheless, research in this field can be useful in the formal analysis of several problems.

I. ANDERSON AND BELNAP'S ENTAILMENT

There are not many books in which systems of alternative logics are defended with the passion and vehemence shown in Anderson and Belnap's *Entailment.* In many passages –of subtle humor– the book seems to be advocating a true crusade against terrible logical heresies rather than to a mere technical discussion on logical-philosophical topics. A & B's divergence from the points of view of *standard* logic (alluded to as "*the Official view*" and "*the opposition*" [to them, of course]) consists in that they understand and characterize in another way the key notion of *logical deducibility*. Both of them mantain that this relation has two basic propierties: *necessity and relevance*. But logicians have abandoned this last requisite in accepting the validity of such inferences as 'it rains and it does not rain; therefore, the moon is made of cheese', whose premise is not relevant for the conclusion, in the sense that (approximately) there is no meaning connection between

[2] I shall quote volume I of this work with the simple abbreviation '*A & B*', which I shall occasionally use to refer to the authors. I give complete references of the works cited in the Bibliography at the end.

them[3]. A & B define *entailment* (a relation they symbolize with '→') as the converse of deducibility and propose to characterize this *notion* formally in the sense in which C.I. Lewis tried to characterize *logical implication* formally with systems like S_4 or S_5. A great deal of A & B's book is devoted to the construction of a formal system suitable for the characterization of a concept of *entailment* imbued with the two previously mentioned characteristics of *necessity* and *relevance*. The final product of those efforts is system *E*. This system is gradually constructed. Chapter I (pp. 3-106) deals with "the pure calculus of *entailment*" (the fragment of *E* whose formulas have as sole connective '→') obtained, basically, through the fusion of two logical systems: the pure implicational fragment of Lewis' S_4, and *R*, a calculus equivalent to certain logical systems studied before by Moh and Church. The reason for the choice of S_4 and *R* as starting points is that, according to A & B, the first system is useful for formalizing the notion of *necessity*, and the second one is useful for characterizing *relevance* (more exactly: they are useful for analizing the formal laws of conditional connectives whose meanings have, respectively, the nuances of necessity or relevance). In chapter II (pp. 107-49) the study is broadened, and formulas containing '∼' and '→' are considered. In the next chapter formulas with the structure '$A \rightarrow B$', where '*A*' and '*B*' pertain to truth-functional propositional logic are investigated (pp. 150-230). The construction of *E* finalizes in chapter IV, where all the formulas that can be constructed with '→' and the usual connectives are considered. Afterwards, a chapter is devoted to neighbor systems of *E* and an appendix to logical-grammatical questions. The treatment that *A & B* give to *E*'s construction is not purely formal. As they progress, they justify the different formal decisions adopted, pointing out the intuitive exigencies leading to them, and discussing

3 The other requirement –necessity– is best accepted. Nevertheless, it is interesting to observe that the texts of logic often give two different definitions of the notion of *validity* and that only one of them conveys note of a necessity. I deal with this in full detail in section 2 of my work 'La lógica formal: su naturaleza y límites'.

philosophically those intuitions. In different passages of the book, for instance philosophical arguments are offered to justify the choice of S_4 over S_5 in characterizing the notion of necessity. A & B's research is completed with metalogical results, comparisions with other systems, etc.

By means of *E*, *A* & *B* seek to provide general formal laws of *entailment* (e. g. '$(A\rightarrow B)\rightarrow((B\rightarrow C)\rightarrow(A\rightarrow C))$'), and laws to indicate the deductive relations that hold between formulas of the usual propositional logic (e. g. '$A\ \&\ B \rightarrow A$') too. But due to the supposition that there is no deducibility without relevance –lacking in the usual logical theories–, *E* contains a revision of the list of inferences admitted in the logic of truth functions. This revision is rather drastic in many respects: venerable rules likc *modus ponens, disjunctive syllogism,* etc., fall. To justify this deep reform of the existent logic, *A* & *B* devote many pages to attacking fiercely the opinion –contrary to the leading intuition of the book– that relevance is not a necessary condition of *entailment*. The attacks are of diverse kinds, from '*ad hominem* jokes at the expense of the opposition' (*A* & *B*, p. xxii), to more elaborated logical or philosophical arguments tending to show that always, when lacking relevance, we are facing an inadmisible deduction. Naturally, the usual definition of deduction is not used. What they try to show is that, somehow, that definition is defective because of its failure to capture a certain requisite demanded by our intuitive, pre-systematic concept of deduction. The main arguments are scattered over the book and brought up wherever a related technical topic is treated. I shall consider in detail those arguments in the next section.

II. THE ARGUMENTS OF ANDERSON AND BELNAP

1. *The argumentation based on intuitive rejection*

The argument to which *A* & *B* more often turn to persuade the reader that deducibility demands relevance is simply the appeal to the intuition that there is no real deduction when the premise[4] is not relevant to the conclusion. This intuitive rejection is often used as evidence throughout the book. I will give a few examples and afterwards make some comments on the value of this kind of argumentation.

(i) According to *A & B* (p. 5), $FH\rightarrow$, a system equivalent to the pure implicational fragment of Heyting's intuitionistic logic, formalizes some traits of *entailment*. Nevertheless, that system contains the theorem '$A\rightarrow.B\rightarrow A$', of which *A & B* affirm that:

> But here we come upon a theorem wich shocks our intuitions (at least our untutored intuitions), (. . .), the principle seems outrageous –such at least is almost certain to be the initial reaction to the theorem, as anyone who has taught elementary logic very well knows (p. 12).

If we read '$\rightarrow$' in the sense of *entailment,* it can be interpretated that the rejection of the theorem is based on considerations of relevance: the theorem would be wrong because it is not intuitively acceptable that from a statement A it could be deduced that any statement B (possibly 'thematically unconnected' from A) permits us to deduce A. This example presents nevertheless another complication, namely that the theorem can be rejected without making any reference to relevance: it can be shown that it does not cope with the demand of necessity imposed on *entailment.*[5] *A & B* recognize this, but they think that our intuition –or that of the apprentice in logic– rejects the theorem both for reasons of necessity and of relevance (see comment on p. 14, *supra*). To show that ordinary intuitions consider relevance as a necessary condition of *entailment,* it would be convenient to give an

4 The grammar of '$\rightarrow$' asks for one-premise inferences.

5 That is, we have no guarantee that if the antecedent of '$A \rightarrow. B \rightarrow A$' is true, then its consequent *must necessarily be so* too. Logicians admit that from a necessary statement we can deduce only necessary statements. Then, replacing 'A' for a contingently true statement and 'B' for a necessary statement, the formula will have a true antecedent and a false consequent.

example is which the rejection of a certain formula could solely be attributed to questions of relevance. With this purpose in mind, *A* & *B* analize '$A \rightarrow .B \rightarrow B$', a formula resisted also by the freshman and clearly objectionable because of considerations of relevance (it is easy to give examples of this formula in which the antecedent has no meaning connection with the consequent), but without parallel difficulties in relation with the requisite of necessity[6]. *A* & *B*'s comments on the two examples cited above show that they consider that intuitive rejections like those illustrated, provide grounds against the validity of a formula.

(ii) On p. 329, a dialogue is imagined between an "Officer" (*O*) and an "Advocate of relevance" (*E*). *O* rejects the conditional 'If every signal has a maximum velocity, then there is a maximum velocity which no signal exceeds'. *E* makes him note that if the conditional is interpreted as 'material', *O* should accept it since he admits the truth of the theory of relativity and, therefore, the truth of the consequent. *O* makes clear that what he meant was that 'there was no way of arguing correctly from the antecedent to the consequent'. *E* suggests then that, when trying to make such a deduction, we would be able to use, added to the antecedent, other relevant premises. *O* assents and *E* gives the death blow: using also Einstein's theory we *can* deduce the consequent from the antecedent. But that is possible only if we are using the "official" theory of deduction. If relevance restrictions are imposed, the deduction will not be legitimate because, in the analized inference, the consequent is deduced from the antecedent and other premises (extracted from the theory of relativity), but the antecedent itself is superfluous, since really only the premises added are used in the process. In that case, one of the premises used is irrelevant to the conclusion and that invalidates the reasoning from the point of view of relevant logic. Because of this, *O* has difficulties to justify his intuition that the conditional is false: 'material' implication

[6] Since '$B \rightarrow B$' *cannot* be false, it is trivially true that if the antecedent of '$A \rightarrow . B \rightarrow B$' is true, its consequent *must* be so too.

would make it true and, if he appeals to an 'enthimematic-deductive' interpretation, according to which a conditional is true when its consequent is deduced from the antecedent with the possible aggregate of other true premises, such interpretation will lead him again to the verification of the conditional, in view of the "official" logic to which he adheres[7] (and his acceptance of the truth of the theory of relativity). The conduct of O in the face of this logical tight corner is disappointing: two phrases later he is heard saying, 'I think I hear my mother calling me' and exits. The moral A & B seem to draw from this is that his logical intuitions are incompatible with a theory of deduction without relevance. Certain intuitions about conditionals are used, then, to support the thesis of relevant logic.

(iii) In many passages of the book one of the stronger intuitions on which they base themselves is cited: the resolute rejection provoked by the inference A & $\bar{A}$ / B, in view of the total lack of connection of meaning between its premise and its conclusion (see, for instance, pp. 151 and ff.). The existence of this strong intuition among laymen is undisputable, and very well known to those who teach elementary logic –to refer to a ground to which A & B often resort to make a judgement.

How can one evaluate the argumentative force of these appeals to ordinary intuitions? I will not fall into the extrem position of denying them relevance. The truth is that intuitions are irreplaceable. Every construction of a logical system consists in the formal elaboration of intuitions. Neither will I deny the existence of intuitions to which A & B appeal. On the contrary, I think that many of them –especially the one cited in (iii) – are widely felt. Nonetheless, in what follows I will try to indicate some reasons why – in the case we are studying– the intuitions resorted to do not have the weight attributed to them.

[7] In fact, Faris has shown this 'enthimematic-deductive' interpretation of the conditional to be equivalent to giving it the truth conditions of the material conditional, if we use ordinary logic (see further on, section IV).

I shall note first that the logical intuitions of the layman differ considerably in their resistance to systematic argumentation. The student who begins studies in logic often carries with him many intuitive convictions, some of which are in conflict with the current logical doctrines. But when the conflict appears, many of the previous convictions weaken and disappear: the student is *convinced,* by the reasons presented to him, that some of his previous beliefs were *false.* Some other times a previous intuition is very strong and the elements of *standard* logic are not sufficient to erradicate it. Two examples are appropriate. Generally, students have intuitions favorable to the validity of forms of inference like the negation of the antecedent and the affirmation of the consequent; nevertheless, they admit that they were mistaken when offered the usual explanations. The previous intuition does not resist the theoretical argumentation. Some other times the contrary happens. The intuition that the falsity of an antecedent on its own does not render false a conditional of everyday language, rarely disappears from the minds of the more acute students with the usual logical argumentations. I believe this difference between more or less resistence to usual logical argumentation is important to evaluate the probatory force of certain intuitions. Scientific research, in all fields, often leads to conclusions not previously obvious, and many of them contradict previous convictions. It is, then, usual for a person to abandon common sense convictions if he considers, more or less consciously, that the evidence provided by some research is more compelling, or more convincing, than the intuitive reasons that previously guided him. I think that it is perfectly natural to say that the intuitions on which *A & B* are basing their case should be considered conclusive evidence against the prevalent logic only if they can effectively resist the argumentations of such logical theory. If, on the contrary, such intuitions decline in front of a contrary logical argumentation, their probatory force will be seriously diminished. Now, I think this is what happens with respect to the principal intuitions on which *A & B*'s arguments are based. In (iii) I pointed out the existence of a strong intuitive rejection many times alluded to by A & B, of the form of inference A & $\bar{A}$ / B. But I will expound in section II.3 a well known argument of Lewis that proves the validity of this from starting from logical rules of an overwhelming obviousness. Experiences with students of logic –that A & B usually consider to be completely relevant– seem to indicate that, when confronted with this argument, the students usually have their intuitions overhauled and accept the existence of perfectly natural forms of inference whose reiterated use leads to non obvious consequences. Students seem to accept willingly that not every "logical fact" is obvious, and that many are accepted simply because of being the *consequence* of *other* obvious facts. To tip the scales in favour of A & B's intuitions it would be necessary to show that the intuitions on which the Lewis argument is based are weaker, or that the argument has some detectable theoretical fault. I will postpone this discussion until section II.3. My intention in this section was only to indicate my argumentative strategy against the intuitive evidence exhibited by A & B. Briefly, the strategy will be this: I accept the existence and the relevance of the intuitions mentioned by A & B; but I will allege that they shock other stronger and more basic intuitions, and I will point out also theoretical reasons of another kind to give preeminence to these.

2. *The criterion of the editor of a mathematical journal*

A & B propose in pp. 17-8 a mental experiment:

> Imagine, if you can, a situation as follows. A mathematician writes a paper on Banach spaces, and after proving a couple of theorems he concludes with a conjecture. As a footnote to the conjecture, he writes: 'In addition to its intrinsic interest, this conjecture has connections with other parts of mathematics (. . .) For example, if the conjecture is true, then the first order functional calculus is complete; whereas if it is false, then it implies that Fermat's last conjecture is correct.'

What would the editor of a mathematical journal do if a paper like this were submitted to him? In *A* & *B*'s story the editor makes it clear that, although the article is acceptable, the footnote is not and will not be published. I have no objection so far. I do not doubt that this would happen in such a situation. But, what reasons would the editor have to refuse the footnote publication? Let us see what A & B have to say.

The mathematician in the story tries to justify the conclusions of his footnote saying, more or less, this: the first order functional calculus is complete, and *it is necessarily so;* therefore, anything implies this fact –particulary the proposed conjecture. It is clear in the context that this 'implies' (like the 'if. . . then' of the quotation) has the meaning of logical implication. If A is a statement of the completeness theorem, and *B* is the mathematician's conjecture, his reasoning is an follows: if *A* is necessarily true, it can be inferred that any statement implies it; in particular, *B* implies *A*. (It is not necessary to analyze the second conditional in the mathematician's footnote for the analysis of *A* & *B*'s opinions, nor for the analysis of my criticism. Remarks similar to those we are going to make with respect to the first conditional apply to it; we only have to analize the idea that what is *impossible* implies anything).

A & *B* think that the footnote would not be accepted by the editor because he would consider the conclusion 'If *B* then *A*' to be *false* with the meaning accorded to the 'If. . . then'. If would be false because without relevance there is no deducibility and therefore B does not imply *A*. The reasoning that led to that conclusion would be a fallacy of relevance. The first conditional in the footnote of the mathematician would not be turned down because of being trivially true, or because of being uninteresting to a mathematician; it would be strictly false.

The context suggests that *A* & *B* do not use this story as a mere illustration; it seems rather to be that through this story, they try to show a fact favorable to their thesis. It would be briefly this: mathematicians in fact do not publish –knowingly, at least– affirmations of logical implication whose antecedent has a clumsy irrelevance with respect to the consequent. A mental experiment like this one plausibly convinces us that an editor would refuse to publish something of this kind. This fact suggests that *relevance* of a logical implication affirmation is a necessary condition for the mathematician with regard to the *acceptability* of something for possible publication. I completely agree with *A* & *B* so far. But they sustain that this inadmissibility is based on the detected falsity of a statement. *A* & *B* do not justify this interpretation of the refusal; besides, there is evidence against it which is favorable to an alternative interpretation. Let us see.

First, an embarrassing circumstance must be pointed out: using the logic ussually used by the mathematician, if *A* is mathematically provable (like in the example) then *it is true* that it is logically implied by *B*. Usual rules inevitably lead to that conclusion. Nowadays, mathematicians often make explicit the methods of proof they use, and some of these —e.g. the unrestricted use of proofs by conditionalization— lead to the alluded result, and support the *truth* of the conditional rejected in the example.[8] The interpretation that the rejection is due to an imputation of falsity can, of course, be held —like *A* & *B* do—, at the price of supposing an inconsistent behaviour in the mathematician (rejecting as false, conditionals whose truth can be proved with the methods of proof he explicitly accepts); a more natural explanation would be preferable. Such an explanation exists and is strengthened through the consideration of some "neutral" examples. We have only to suppose that, to be acceptable for publication, a mathematical affirmation needs not only to be true, but also to have a non trivial interest. In other words: we should suppose that the mathematician rejects (for publication) not only falsities, but also stupid

[8] '$B \rightarrow A$' is obtained through conditionalization as follows: Let us formulate A by means of the conditional 'L is a (here comes an adequate description of elemental functional calculus) $\supset$ *L* is complete' which we shall abbreviate '$A_1 \supset A_2$'. A_2 can be deduced from A_1 (in symbols: $A_1 \vdash A_2$) with the help of the completeness theorem. According to usual logic, deduction is preserved if we extend the set of premises, and it holds that $B, A_1 \vdash A_2$. By conditionalization we obtain $B \vdash A_1 \supset A_2$, which expresses the same as '$B \rightarrow A$'.

truths.[9] This is obvious, as is also obvious that conditionals like that of the example lack of any interest. This provides us with an explanation of the fact described by *A* & *B* which does not require us to suppose the implicit adherence of the mathematicians to the postulates of a relevant logic. A neutral example will strengthten these considerations. *A* & *B* consider that there is an extensional sense of 'or' that satisfies the rule of addition: *A* / *A* or *B*. Therefore, if a mathematician infers from an interesting theorem C on Banach spaces the affirmation that '*C* or 2+2=5', his conclusion will be correct. If he then affirms the conditional 'If *C* then *C* or 2+2=5' –using 'if. . . then. . .' in the sense of logical implication– he will say something true too, *from the point of view of A & B's alternative logic* (see *A* & *B*, p. 232). The reader can imagine nonetheless what would happen to this mathematician if he intends to publish these kind of conclusions. They will be of course rejected as those in *A* & *B*'s example were. But *A* & *B* cannot explain the rejection now saying that the conditional from the last example is false, because it is true even *for the relevant logic they defend.* The explanation should be sought in the lack of interest of the conditional, and pragmatic considerations of this kind. It is clear, therefore, that very similar rejections to the one in *A* & *B's* example, are based on motives different from the charge of falsity. Thus, if we explain in this way what happens in the previous example, we will avoid the supposition that the mathematician has a strange incoherence, we will give a very natural explanation of what happens in the mental experiment described, and. . . we will disqualify one of *A* & *B*'s arguments.

3. *Criticism of Lewis' argument*

Arguments trying to show that, if certain principles of inference are accepted an irrelevant proposition can be deduced

[9] In fact, mathematicians employ "stupid" truths in their texts, e.g. as auxiliary steps in a proof; but those statements are not presented as "important" conclusions, as the mathematician of the example wanted to present them.

from a contradiction, are known from antiquity. One of them has been recently divulged by Lewis and Langford (*Symbolic Logic*, pp. 248-51). In it are used the following rules of inference:

Simplification (Simp.):	$A \,\&\, B \,/\, A$;	$A \,\&\, B \,/\, B$
Addition (Ad.):	$A \,/\, A \vee B$;	
Disjunctive Syllogism (D.S.):	$A \vee B,\ \bar{A} \,/\, B$;	

With the rules mentioned we can arrive at an arbitrary statement, starting from $A \,\&\, \bar{A}$. The proof –from now on 'Lewis' argument'– is as follows:

1. $A \,\&\, \sim A$	(Premise)
2. A	(1, Simp.)
3. $\sim A$	(1, Simp.)
4. $A \vee B$	(2, Ad.)
5. B	(3, 4, D.S.)

Lewis' argument is very interesting because it shows an unexpected and anti-intuitive consequence of the reiterated use of a few intuitive rules. There can be hardly any doubts about the intuitive character of the rules. Simplification and Disjunctive Syllogism are both very obvious and commonly used. Only Addition raises some doubts in elementary logic courses, but when you try to remove them, they turn out to be doubts on the usefulness of the rule rather than doubts on its validity. In any case, the rule is so obvious in many applications that we use it without consciously realizing it, like when we apply a theorem of the form '$a \leqslant b \rightarrow \ldots$' to a case in which $a=b$. Now, if it is accepted that the inference procedures used in the argument exemplify natural forms of reasoning, and that the reasonings they validate constitute "good deductions" in a pre-technical sense, a powerful evidence against the theses of relevant logic will be at hand: there could be *deducibility* without *relevance* in an intuitive sense of the first notion. It could be interpreted that our intuitive ideas on deduction, and some rules in agreement with them, have consequences of which we are not usually conscious, and that would explain simultanously the existence of "irrelevant" deductions and our pre-systematic conviction that such a thing is not possible. Naturally, the followers of relevant logic could have a defense against this if they could show Lewis' argument to be defective in any way. This is exactly what *A* & *B* try to prove. In what follows I will deal with their objections; I will analize in Section III other possible attacks on the argument from the point of view of other authors.

A & *B*'s criticism is a little twisted, consisting basically of attacks on the irrestricted validity of Disjunctive Syllogism. According to them, (pp. 165-7), 'or' has often the purely extensional sense usually attributed to it by logicians. But its sense has sometimes intensional nuances and, then, relevance between disjuncts is necessary for the truth of the disjunction. The rute of Addition would be legitimate only with the 'or' being extensional. This is plausible: if in certain examples 'or' is used to indicate, among other things, relevance between the propositions connected, in that case the proposition *A* or *B* cannot be inferred from *A*, for any arbitrary *B*, since *A*'s being true is no warranty of *A*'s being relevant for any proposition whatsoever. Disjunctive Syllogism would be valid only with an intensional 'or'. Then, the error of the argument considered by us would consist in a fallacy of ambiguity: to obtain 4 from 2, '$\vee$' must have the usual extensional sense; but to obtain 5 from 3 and 4, '$\vee$' should be replaced by an intensional 'or'. For the complete deduction to be valid, 4 would have to be understood in two steps of the argument with divergent senses.

A & *B*'s criticism is clearly outlined; but, why does Disjunctive Syllogism ask for an intensional 'or'? *A* & *B* are far from having given convincing reasons for this. When they first state their objection (§ 16.1, pp. 163-7), they limit themselves to referring the reader to three ulterior sections (§ 16.3, § 27.1.4. and § 16.2.2) without giving there *any* argument to support the D.S. restriction they believe to be necessary. Only in the first of the three sections do they try to argue against D.S. used with extensional 'or', but their closing phrase is '(...) we do *not* claim that our argument on this point, such as it is, is conclusive' (p. 177, *A & B*'s underlining). The reservation is justified. The argument alluded to by the quotation consists in (1) observing that *some* disjunctions supporting D. S. have an intensional character, noticeable because of supporting (possibly counterfactual) subjunctive conditionals, whose antecedent is the negation of one disjunct and whose consequent is the other disjunct; (2) exhorting the reader to find a disjunction supporting D. S. and *not supporting* a subjunctive conditional like the one depicted. The hypothesis that D. S. is only valid when read with a non-extensional 'or', is guaranteed if the reader can not find such an example.

The weakness of the argument –confessed in the quotation– is obvious. *A & B* have *no* general argument against the existence of legitimate cases of D. S. with extensional 'or'. They only show valid cases of intensional disjunction and exhort the reader to find valid cases in which the disjunction does not support a subjunctive conditional, hoping the reader will fail. I will not only point out that *A & B* are not conclusive; in what follows I will respond to the exhortation. To do this I will adapt some interesting examples of conditionals studied by Ernest Adams (see Bibliography). Let us consider the disjunction.

(1) Oswald killed Kennedy or somebody else did it.

Let us imagine somebody who firmly believes that Kennedy was killed by a lonely killer, although he is not sure that the killer was Oswald. This conviction will make him believe that (1) is true. Let us suppose now that this person comes to admit also, due to additional evidence, that

(2) Oswald did not kill Kennedy.

I submit to the reader's intuition the following question: having these two premises, the person of our example will deduce

(3) Somebody else (*i.e.*, in context, somebody who was not Oswald) killed Kennedy?

I have no doubt what the reader will answer. Let us ask ourselves now if the person who believed in (1) ought to admit that

(4) Had Not Oswald killed Kennedy, somebody else would had done it.

Of course not. In order to believe (1), the conviction that *somebody* killed Kennedy is enough. To believe (4), you have to support some sort of theory of historic necessity or inevitability,[10] or, at least, to suppose that Kennedy was a victim of the conspiracy of several persons, any of which could be replaced by another of them, if necessary. But obviously there are clusters of coherent believes that would endorse (1) but not (4). We have, then, a non-trivial example that clearly shows the possibility of a disjunction supporting D. S. without entailing any subjunctive conditional of the kind described before. Therefore, A & B's conjecture in the sense that the inference 'A or B, $\sim A$ / B' is admissible if from 'A or B' it can also be derived 'if it were not the case that A, then it would be the case that B', is false. And this conjecture was all A & B had to offer against irrestrict D. S. and Lewis' argument.

Some final comments on the usefulness of the example adapted from Adams, and some theoretical considerations to stregthen it, can be of some interest. It is not easy to refute A & B's conjecture mentioned at the end of the last paragraph, clearly enough. To do it, it is necessary to find a case of 'A or B' that does *not* imply a subjunctive conditional; but 'A or B' *always* implies some kind of conditional (if you accept 'A or B', then you accept that *in some sense* it is true that 'if A is not true, then B is true'). And, as it is generally

[10] Gladys Palau, explains in a recent paper, the meaning that could be given to (4) (see Bibliography).

not clear if a conditional has subjunctive connotations or not, it can be very difficult to be sure a certain case of 'A or B' has not subjunctive consequences. That is why Adams' analyses are a considerable help at this point. Adams found examples where it is clear that certain conditionals formulated in the indicative do not imply the corresponding subjunctive conditional. Adapting one of his examples (through the transformation of one of the indicative conditionals into disjunction) we have found a case of 'A or B' which clearly *does not imply* a subjunctive conditional (although it does imply a certain indicative conditional[11]); since it is equally obvious that such a case of 'A or B' admits of the application of D. S., we have refuted the so-called need for a restriction of this rule, as suggested by A & B. With the help of some recent theories on subjunctive and counterfactual conditionals, a theoretical explanation of why 'A or B', supplemented with '$\sim A$', implies 'B', in cases when from 'A or B' alone 'if it were not the case that A then it would be the case that B' does not follow, can be given.[12] Although this explanation is not necessary for our analysis –nor do I want to rest on it for my argumentation since the theories I am going to mention are not beyond controversy–, it will clarify a bit more the logical questions analyzed. In broad outline, it can be said that, for 'A or B, $\sim A$ / B' to be valid it is only necessary that 'A or B' say something *about the real world:* that in that world either occurs alternative A or occurs alternative B. This affirmation about the real world, complemented with the affirmation that the first alternative does not take place, is enough to justify the conclusion, since it refers to the real world too (if I know that in the real world one of the alternatives takes

[11] The statement (1) implies 'If Oswald did not kill Kennedy, somebody else did it'; but there is a strong contrast in ordinary language between this affirmation and sentence (4).

[12] By the metatheorem of deduction, if 'A or B, $\sim A$ / B' is valid, then 'A or B / $\sim A \supset B$' is valid too. Gladys Palau has pointed out to me that A & B's conjecture amounts to saying that, at least in this case, the '$\supset$' in the conclusion of the metatheorem can be replaced by a subjunctive conditional. A & B give no arguments in favor of the metatheorem so understood, and I show in the text that in fact it would be false if it were formulated thus.

place, but that it is not the first one, I can conclude that *in that world* the other one takes place I. But if 'A or B' refers only to the real world, we cannot derive from it that 'if it were not the case that A then it would be the case that B', because, according to the most recent theories, a subjunctive conditional of this kind not only says something about the real world: it says something, too, about what happens in certain possible worlds. Therefore, in the semantics of theories like those of Stalnaker and David Lewis (see Bibliography), we can build models in which 'A or B' is true in the real world and 'if it were not the case that A it would be the case that B' is false because the disjunction is not fulfilled in other possible worlds. (Such a model is obtained in Stalnaker's theory with these two assumptions: (i) In the real world it is the case that A but not B; (ii) in the possible world more similar to the actual one where it is not the case that A, neither is it the case that B. 'A or B' is true by (i); 'if it were not the case that A it would be the case that B' is false by (ii))[13]. The analyses of subjunctive and counterfactual conditionals with the help of a semantic of possible worlds give an idea of why D. S. can be valid even with a disjunction which does not support a subjunctive conditional.

I said before that only in section 16.3, A & B present arguments against D. S.; really, in 16.2.2 they add an 'independent proof' of D. S.'s general invalidity. But in the general context of the discussion, such 'proof' has to be considered either a joke or a *petitio principii.* They key paragraph of the last section quoted says:

> (. . .) though $\bar{A}(A \vee B) \rightarrow B$ holds for special cases, it does not hold in general, for (here comes the "independent proof" promised in 16.1), $\bar{A}(A \vee B) \rightarrow B$ iff $\bar{A}A \vee \bar{A}B \rightarrow B$, only if $\bar{A}A \rightarrow B$, which is absurd. (p. 174).

[13] It is a little bit more complicated to construct a model of this kind within the theory of Lewis, because he has good reasons for not adopting a unicity assumption that simplifies Stalnaker's layout (see Lewis, *Counterfactuals,* pp. 77-83).

D. S. is refuted in this quotation assuming that Lewis' argument is invalid (since it precisely establishes that $\bar{A}A \to B$). Of course, if you try to object to D.S. in order to attack Lewis' argument, you cannot do so assuming the last one to be invalid.

So, A & B's criticism of Lewis' argument is entirely inadequate. On the other hand, the "opposition" has an excellent argument in favor of the thesis that there can be deduction without relevance. A & B's situation is made worse since they give great importance to intuitions and Lewis' argument consists of very intuitive steps. A & B could try this defense: it is true that there are intuitions that support the rules used in Lewis' argument, but there are also some against the validity of $A \ \& \ \sim A. \to B$ (see *supra,* II.1. (iii)), and then we have a clash of intuitions rather than a one sided intuitive support to Lewis. I think that this defense would be all right but I want to end this analysis pointing out two important reasons why I believe that in this clash of intuitions priority should be given to those supporting Lewis' rules. First, these last intuitions are more basic: they deal with very simple and common principles of inference. Simplification could not be more obvious; D. S. corresponds to the well-known method of 'reasoning by elimination'. Addition –as I remarked before– is automatically applied in Mathematics. On the other hand, intuitions against $A \ \& \ \sim A. \to B$ deal with what happens in a logical situation in which laymen are never involved (who reasons from explicit contradictions?) and much more complex that the basic inferencial cases seen before. I do not pretend to be conclusive about intuitions; I only try to persuade the reader that if my intuitions on *simple, frequently-used* rules clash with intuitions about *more complex* and *less familiar,* situations it seems reasonable, *prima facie* to keep the first ones (because it seems to be easier to be wrong with respect to the last ones). Because of theoretical reasons it could be better to give up the first ones rather than the second ones; but such reasons *must* exist, and they ought to be important and convincing. These are the reasons I have unsuccesfully looked for in A & B's arguments. Besides this argument concerning the more basic character of the intuitions supporting Lewis' argument[14], there is a stronger theoretical-pragmatic reason for giving priority to those intuitions: to accept *the others* leads to such a drastic revision of every logical theory available that excellent arguments would be required to justify such a decision.

4. *An inconsistency in the "official" use of 'it follows that'*

A & *B* quote, on page 332, the following remark of Kleene (from *Introduction to Metamathematics*, p. 532), on Quine's *Mathematical Logic:*

> See Rosser 1942 and Quine 1941, concerning the fact that the Burali-Forti paradox arises in the system of this book (although Cantor's paradox apparently is avoided)...

After quoting this, *A* & *B* invite the reader to speculate on the question as to how we can get, in the system discussed, one paradox and not the other, *from the point of view of classical deduction,* which admits of $A \mathbin{\&} \bar{A}. \rightarrow B$ (and the same would hold with regard to intuitionistic logic). The remark is excellent. It points to an inconsistency in the use of the notion of *deducibility,* since Kleene's affirmation is untenable using the classical or intuitionist notion of deducibility. The inconsistency seems to show that Kleene reasoned at one time, as if he accepted the thesis that there is no deduction without relevance, without noticing it. *Granting this,* what the quotation affirms can be maintained: if we restrict ourselves to "relevant" modes of inference it *is* possible to deduce one paradox and not the other –unless, of course, in the system of *Mathematical Logic* (lst ed.) there is an unknown, "direct" proof of Cantor's paradox, but we have the hypothesis in the discussion that there is no such thing.

[14] Teaching experiences like those described briefly at the end of II.1 give additional support to the idea that Lewis' intuitions have a more basic character than the intuition against '$A \mathbin{\&} {\sim}A. \rightarrow B$'.

The slip is meaningful and we will come back to it later (section IV), but I will point out here that it gives no support to the theses of *A* & *B*. And this is so because the author quoted —or any other caught in a similar embarrassing situation— can always correct himself, accepting that whenever one paradox can be deduced, so can the other, and making clear that what he really meant can be better expressed if we talk of "direct" proofs of the paradoxes, or something like that.

5. *Critical evaluation*

We have seen that the arguments considered in II.2 and II.4 are not conclusive: *A* & *B*'s evidence there analyzed, can easily be reinterpreted in a way not favorable to the theses of those authors. The criticism of Lewis, studied in II.3, lacks adequate foundations and bases itself essentially on a false logical conjecture. With respect to the intuitions in favour of relevant logic, analyzed in II.1 we have noticed that one of the strongest —(iii)— shocks other intuitions we have judged to be more worthy of consideration for various reasons (end of II.3). The intuition against '$A \rightarrow . B \rightarrow B$' (II.1. (i)) is only indirectly affected by the analysis of II.3. Reflections on necessity tend to support this formula (see note 6); the only reason to reject it is the conviction that it can not be deduction if the premise 'has nothing to do' with the conclusion. But the intuitions supporting Lewis' argument are against this conviction, and therefore against the only evidence alleged against the formula. (The case of '$A \rightarrow . B \rightarrow A$' is less important, since, as we have already seen, it can also be rejected because of "classical" criterions). With respect to the intuitions depicted in II.1. (ii), that also lead to a clash with theses of classical logic, they are not really incompatible with the existence of deduction without relevance: in effect, in that case the "classical" theses that shock the intuition are a combination of theses on conditionals and theses on deducibility, and it could be argued that the first are mistaken and not the others (if we decide to accept the intuitive evidence in this example).

This evaluation shows that A & B are far from having conclusively argued againt the classical analysis of deducibility. Their position is all the more jeopardized if we remember that they are forced to give a specially conclusive argumentation because of the drastic and deep reform of logical theory demanded by their criticisms.[15]

There is another way of facing A & B's criticisms, which I have not thought necessary to use. It is to observe that, even if the intuitive notion of deducibility asks for relevance, the employment of a technical concept, like that of classical logical theory, in which such a requisite is not indispensable, could be very useful. But notice that our defense of Lewis' argument leads to a more drastic rejection of A & B's criticism: that argument would show that even with *intuitively accepted* forms of deductive inference, "irrelevant" reasonings could be justified. Because of this and other reasons, Lewis' argument has a key position in our "counter attack" to the theses on deducibility and relevance. Due to this it is important to consider other possible criticisms of the argument, and this will be the subject of the next section.

[15] J. A. Robles has noticed a certain parallelism between this form of facing A & B's "dissident logic", and the argumentation Hilbert used to oppose Brower when Brower attacked the principle of the excluded middle.

BIBLIOGRAPHY

Adams, Ernest, 'Subjunctive and Indicative Conditionals', *Foundations of Language*, vol. VI 1970, pp. 89-94.

Anderson, Alan Ross y Belnap, Nuel D. *Entailment*, Princeton University Press, Pinceton, vol. I, 1975.

Baker, A. J. '*If* and ⊃', *Mind*, vol. 76, No. 303, July 1967, pp. 437-8.

Bennett, Jonathan, 'Meaning and Implication', *Mind*, vol. 63, No. 252, October 1954, pp. 451-463.

Bunge, Mario, 'La paradoja de la adición: respuesta al maestro Margáin', *Crítica*, vol. VII, No. 20, Octubre 1975, pp. 105-7.

__________'The Paradox of Addition and its Dissolution', *Crítica*, vol. III, No. 9, Septiembre 1969, 27-31.

Duncan-Jones, Austin E. 'Is Strict Implication the same as Entailment?, *Analysis*, vol. 2, 1935, pp. 70-8.

Faris, J.A. *Truth-Functional Logic*, Routledge and Kegan, Paul, London, 1962.

Haack, Susan, *Deviant Logic*, Cambridge University Press, London, 1974.

__________ *Philosophy of Logics*, Cambridge University Press, Cambridge, 1978.

Kleene, Stephen C. *Introduction to Metamathematics*, D. Van Nostrand Company, 1952.

Lewis, C.I. y Langford, Cooper, *Symbolic Logic*, 2a. ed., Dover, New York, 1959.

Lewis, David, *Counterfactuals*, Basil Blackwell, Oxford, 1973.

Margáin, Hugo, "La paradoja del Dr. Bunge", *Crítica*, vol. VI, No. 18, Septiembre 1972, pp. 113-6.

__________"Validez, Inferencia e Implicaturas I", *Crítica*, vol. VIII, No. 23, Agosto 1976, pp. 63-98.

Meadows, Dennis (et. al.) *The Limits to Growth*, Universe Books, New York, 1972.

Nelson, Everett J. "Intensional Relations", *Minds*, vol. 39, 1930, pp. 440-53.

__________"On Three Logical Principles in Intension", *The Monist*, vol. 43, 1933, pp. 268-84.

Orayen, Raúl, *La lógica formal: su naturaleza y límites*, Publicaciones de la Facultad de Humanidades, Universidad Nacional del Comahue, Neuquén, Argentina, 1982.

Palau, Gladys, "Condicionales contrafácticos: condiciones de verdad y semántica de mundos posibles", *Crítica*, vol. XII, Abril 1980.

Robles, José A. "Comentarios en torno a Bunge, Margáin y la paradoja", *Crítica*, vol. VIII, No. 23, Agosto 1976, pp. 105-13.

Russell, L. J. "If and ⊃", *Mind*, vol. 79, No. 313, January 1970, pp. 135-6.

Smiley, T. J. "Entailment and Deducibility", *Proceedings of the Aristotelian Society*, vol. 59, pp. 233-54.

Stalnaker, Robert C. "A Theory of Conditionals", in N. Rescher (ed.), *Studies in Logical Theory*, Blackwell, Oxford, 1968.

Stalnaker, R. C. y Thomason, R. H. "A Semantic Analysis of Conditional Logic", *Theoria*, vol. 36, 1970, pp. 23-42.

Strawson, Peter, *Introduction to Logical Theory*, Methuen & Co., London, 1952. Review of Von Wright, *Logical Studies* (Routledge and Kegan Paul, Londres, 1957), Appeared in *The Philosophical Quarterly*, vol. 8, 1958, pp. 372-6.

Von Wright, G. H. "The Concept of Entailment", in Von Wright, *Logical Studies*, Routledge and Kegan Paul, London, 1957.

Wolf, Robert G. "Are Relevant Logics Deviant?", *Philosophia*, vol. 7, No. 2, June 1978, pp. 327-40.

RESUMEN

La lógica clásica (en adelante, 'LC'), considera *deductivamente válido* un razonamiento como "llueve y no llueve; luego, la luna es de queso", a pesar de que no parece haber conexión significativa entre su premisa y su conclusión. Los "lógicos relevantes" sostienen que la noción clásica de *deducibilidad* es defectuosa pues, en un sentido intuitivo importante, no puede haber deducibilidad sin conexión entre los contenidos de las premisas y la conclusión. En consecuencia, tratan de desarrollar sistemas lógicos en los que no se validen razonamientos como el citado antes y se formalice un concepto alternativo de deducibilidad.

En este artículo se examinan críticamente las objeciones de la lógica relevante a la noción de deducibilidad de LC y se defiende la tesis de que tales objeciones están mal fundadas. En la sección final (IV) se analizan el *status* y la utilidad que puedan tener los sistemas formales de lógica relevante.

I. *El* Entailment *de Anderson y Belnap*

Se describe brevemente el contenido de esta obra (en adelante citada mediante la abreviatura '*A* & *B*'), la más elaborada y completa sobre el tema de la lógica relevante.

II. *Los argumentos de Anderson y Belnap*

Se analizan los argumentos diseminados a lo largo de *A* & *B* en contra del concepto clásico de deducibilidad.

1. El elemento de juicio del rechazo intuitivo.

Se citan varios pasajes en que *A* & *B* utilizan en apoyo de sus tesis filosóficas el rechazo intuitivo suscitado por muchas inferencias validadas por LC (por ejemplo, las de la forma A & $-A$ / ∴ B). Se sugiere luego un criterio para evaluar intuiciones "disidentes" de la lógica *estándar:* el valor de tales intuiciones dependerá del grado en que "persistan" ante una argumentación lógica en contrario. Se posterga hasta el parágrafo II.3 el examen de un argumento lógico difundido por C. I. Lewis en contra de las intuiciones que soportan la lógica relevante.

2. El criterio del editor de una revista matemática.

Para hacer plausible las tesis de que los matemáticos consideran incorrectas las inferencias que carecen de relevancia, *A* & *B* imaginan una situación ficticia en que un matemático intenta publicar cierto artículo en una revista especializada. Después de plantear una conjetura *C*, el matemático sostiene en una nota que *C* implica lógicamente el teorema de completitud del cálculo funcional (porque aunque *C* no tiene nada que ver con ese teorema "temáticamente", lo implica –desde el punto de vista de LC– por ser tal teorema una verdad necesaria). *A* & *B* consideran que el editor rehusaría publicar tal nota considerando falso el enunciado de implicación lógica aludido. Se sostiene en este trabajo que si bien es plausible suponer que se producirá tal rechazo, puede interpretarse que las razones podrían ser diferentes a las alegadas por *A* & *B*: un matemático puede negarse a publicar razonamientos torpemente inútiles, aun cuando sean correctos. Se refuerza esta interpretación alternativa dando ejemplos de otras inferencias que también serían rechazadas a pesar de que son correctas aún en el sistema de *A* & *B*.

3. La crítica al argumento de Lewis.

C. I. Lewis ofrece esta demostración en apoyo de la validez del esquema $A \mathbin{\&} -A \mathbin{/} \therefore B$:

1. $A \mathbin{\&} -A$	(premisa)
2. A	(1, Simplificación)
3. $-A$	(1, Simplificación)
4. $A \vee B$	(2, Adición)
5. B	(3, 4, Silogismo Disyuntivo)

A & *B* sólo tienen una conjetura lógica en contra de este argumento: piensan que el Silogismo Disyuntivo sólo es válido si la premisa disyuntiva "*A* o *B*" utiliza un "o" intensional, cuyo carácter se pone de manifiesto porque "*A* o *B*" implica el condicional subjuntivo "si no se diera *A* se daría *B*" (más brevemente: "*A* o *B*, $-A$ / *B*" es válido sólo si de "*A* o *B*" puede derivarse "si no se diera *A*, se daría *B*"). La Adición, en cambio, sólo es válida con un "o" extensional. En ese caso, el paso de 3 y 4 a 5 exige que el "o" de 4 sea intensional, y el paso de 2 a 4 sólo es legítimo si tal "o" es extensional. El argumento de Lewis sería una falacia de equívoco.

Se refuta la conjetura sobre el Silogismo Disyuntivo, usando estos enunciados:

(1) Oswald mató a Kennedy u otro lo hizo.
(2) Oswald no mató a Kennedy.
(3) Otro lo hizo.
(4) Si Oswald no hubise matado a Kennedy, otro lo hubiese hecho.

Claramente, (1) y (2) implican (3), pero (1) no implica (4), en contra de la conjetura de *A* & *B*. Se refuerza con consideraciones teóricas el uso de este contraejemplo.

El argumento de Lewis se apoya en reglas intuitivas para probar algo que la intuición rechaza, como se vió en II.1. Tenemos pues, un choque de intuiciones. Se argumenta en el trabajo que en este choque se debe dar prioridad a las intuiciones que apoyan el argumento de Lewis, por ser más básicas, y ocuparse de situaciones lógicas más simples y frecuentes. Algunas consideraciones pragmáticas refuerzan esta conclusión.

En el parágrafo II.4 se analiza un argumento de *A* & *B* de menos importancia y en II.5 se hace un balance crítico de la discusión desarrollada en los cuatro parágrafos precedentes.

(En las secciones III y IV, que se publicarán en el próximo número de *Crítica*, se analizan argumentos de otros autores en favor de la lógica relevante y se examinan el *status* filosófico y utilidad de los sistemas formales de lógica relevante.)

[R. O.]