DEDUCIBILITY IMPLIES RELEVANCE? A NEGATIVE ANSWER (II)*
(On the philosophical status of relevant logic)

RAÚL ORAYEN
Instituto de Investigaciones
Filosóficas, UNAM

In section I of this paper I briefly described the contents of *Entailment* by Anderson and Belnap (which I refer to as 'A & B'), where the philosophical thesis that there is not *deduction* –in an intuitive sense of the concept– when there is not *relevance* between the premises and the conclusion of a given argument, is held. Due to this, reasoning schemas such as 'A & –A/B' (validated by the "official" deductive logic) should be considered non-valid, and we would have the theoretical need to construct logical systems (the "relevant" logics) which would allow us to characterize in a more adequate manner the notion of deducibility. In section II, I critically analyzed the arguments that A & B give to support these philosophical theses and I concluded that they lacked adequate grounds. In particular I considered wrong an objection of A & B to a well known argument of Lewis which tries to prove the validity of 'A & –A/B'. The steps and justifications of that argument are the following:

1. A & –A	(Premise)
2. A	(1, Simplification)
3. –A	(1, Simplification)
4. A v B	(2, Addition)
5. B	(3, 4, Disjunctive Syllogism)

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A & B object to this argument because of *the use of Disjunctive Syllogism.* In II.3 I did show that their objection is based in a false logical conjecture. But other authors may oppose Lewis' argument for other reasons, and that is the subject I will study in section III. Finally, in section IV, I will deal with the usefulness and philosophical *status* of the formal systems of relevant logic.

III. OTHER ARGUMENTS AGAINST DEDUCTION WITHOUT RELEVANCE

Many writers, besides A & B, have put forward arguments against deduction without relevance. It is impossible to consider all of them within the dimensions of an article (although it is easy in many cases to adapt my analysis of section II to them, since A & B use many arguments common to the literature on the subject), but the panorama would be more complete if we examine what other attacks can be made on Lewis' argument. Since this argument is an excellent piece of evidence in favor of deduction without relevance, we would have a comprehensive criticism of the points of view of relevant logic if we deal adequately with the different strategies in order to attack that argument. Besides D.S. two other rules are used in Lewis' argument: the Simplification and Addition, and a property of the deducibility relation which can be stated as: if from certain premise(s) it is possible to reach a certain conclusion, through a chain of valid deductive steps, then the inference which can be made directly from the premise(s) to that conclusion would be valid also. All these assumptions have been attacked at sometime by philosophers akin to A & B. Let us see some of their arguments.

1. *Simplification*

In 'Intensional Relations', an article that can be considered a pioneer in the field of relevant logic, E.J. Nelson abandons Simplification. He does so, not in order to attack Lewis' argument (which can be rejected with only the abandonement of Addition, proposed also by Nelson) but to neutralize a proof in which an "irrelevant" argument is validated using Simp. and the Principle of the Antisyllogism (in A & B's notation: $(A\&B \rightarrow C) \leftrightarrow (A\&-C \rightarrow -B)$). The inference is this:

(1) $A\&B \rightarrow A$ (Rule of Simplification)
(2) $A\&-A \rightarrow -B$ (From applying Antisyllogism to (1), replacing 'C' by 'A').

How can we justify the abandonment of Simp.? Nelson rejects that the rule is valid for a conjunction of intensional nature introduced in his article with explanations not altogether clear. When he tries to indicate the meaning of this conjunction he says:

> I do not take pq to mean "p is true and q is true", but simply "p and q", which is a unit or whole, not simply an aggregate, and expresses the *joint* force of p and q (p. 444, author's underlining).

From this conception of conjunction a consequence would follow with respect to inferences of the form "$A\&B \rightarrow C$":

> pq does not entail r, unless both p and q *function together* in entailing r (p. 444, author's underlining).

If the last consequence is accepted then Simplification should be rejected since in typical cases of application of this rule only one of the conjunctives (not both, "functioning together") is used to support the conclusion.

In 'Meaning and Implication', a paper that appeared much later than that of Nelson, Jonathan Bennett remarks that these ideas do not give, in any way, a solution capable of neutralizing arguments like that of Lewis. His arguing is clear and overwhelming. He begins conceding to Nelson (perhaps with excessive kindness) that with his intensional use of conjunction, it seems to be justified to reject Simp. But then he adds:

> But it cannot be denied that there *are* relations corresponding to the extensional conjuntion and disjunction, of material and strict implication, and these are all that are required by the independent proofs of the paradoxes [e.g. Lewis' argument] (p. 454).

Bennett is right. To show that there can be deduction without relevance, it is enough to find a valid argument whose conclusion is irrelevant to the premises. If, in such argument, a rule for conjunction is used, we have to consider when discussing the validity of that rule in that context, the meaning with which conjunction is used *there*. Due to the purely extensional meaning that conjunction has in Lewis' argument, it is not affected by Nelson's doubts which are concerned about Simp. when conjunction is used in another way.

An interesting additional remark of Bennett (pp. 458 and ff.) shows that Simp should be considered valid even in the intensional logic outlined by Nelson. The reason is this: because of the belief that the deducibility is always accompanied by a connection between meanings, Nelson thinks that the *entailment* is "an identity beween a structural part (though not necessarily less than the whole) of the antecedent and the entire consequent". But this suggests that any time that A → B, A can be expressed in an equivalent fashion as the conjunction of B and something else, let us say C, and the entailment A → B would be equivalent to B&C → B, a flagrant case of Simp. I think Bennett is right in this extension of his criticism, but because of reasons pointed out in the past paragraph, these considerations are not needed to defend Lewis' argument.

There is a more general reason why rejection of Simp. cannot provide an adequate defense of the points of view of relevant logic. We have seen that its exponents give much credit to evidence supplied by the "pre-theoretical" logical intuitions (and Nelson is no exception: the appeal to this evidence underlies many of his considerations); in this case they cannot ignore the great amount of intuitive evidence supporting Simp. Because of this, even some writers with ideas similar to those of Nelson, have considered that abandoning Simp. is an heroic *ad hoc* and unnacceptable resort.[16]

2. *Addition*

Of the logical assumptions used in Lewis' argument, this has been the most often attacked by thinkers oriented in the direction of relevant logics (that is why, the criticism of the argument based on the rejection of D.S., which is A & B's strategy, constitutes a rather heterodox line of action). Other systems of relevant logic, different from *E*, have been constructed, in which this rule is dropped (see, for instance a system by Parry, described in A & B, § 29.6.1, pp. 430-2). Nelson, in the paper quoted, abandons Addition too, because his treatment of 'or' is similar to that of 'and': he defines an intensional sense of 'or' and then argues that, used in this way, Addition does not hold for it. Because of analogous reasons to those expounded when discussing Simp., his remarks do not affect Lewis' Argument, where disjunction is used differently.

In section 11, chapter III of *Introduction to Logical Theory* Strawson objects to Addition also. But this section is devoted to a comparison between the ordinary 'or' and 'v', and *prima facie,* Strawson seems to be interested in objecting to Addition only for the first one: on p. 90, *infra,* he points out that the fact that 'v' holds for the law '$p \supset p v q$' shows a difference between its meaning and that of the everyday 'or' in *some* uses. In such uses, 'or' strongly contrasts with 'v', because these uses are cases in which the truth of a disjunct is not sufficient condition for the truth of the disjunction. But then Strawson analyzes *other* usages of 'or', in which it results in being more similar to 'v' and he finds *new* reasons to reject Addition in these cases. Although he is only explicitly occupied with 'or', the fact that he takes into account usages in

[16] For instance, A.E. Duncan-Jones in 'Is Strict Implication the same as Entailment?', where he proposes another solution to Nelson's paradox of the antisyllogism". Nelson himself, in an article posterior to the one quoted in the text, ('Three Logical Principles in Intension') looks for another solution to the paradox not requiring an abandonment of Simp.

which the particle seems to have the same truth conditions that 'v' has, could make one suspect that he could consider his objections valid even when they are applied to the last connective, in which case they would be relevant in evaluating Lewis' argument. Therefore it will be better to point out some defects in his argument (even if taken only in connection with 'or').

Consider the statements:

(1) It was John
(2) Either it was John or it was Robert

Strawson says that (1) *entails* (2) in the sense that it confirms (2) but that it does not *entail* it in the sense that the step from (1) to (2) constitutes a *logically proper step.* The reason for this last denial is that:

> the alternative statement carries the implication of the speaker's uncertainty as to which of the two it was, and this implication is inconsistent with the assertion that it was John. (p. 91)

In his article 'Validez, Inferencia e Implicaturas I', Hugo Margáin shows that Strawson's analysis involves confusions that can be cleared-up with the help of the well-known distinctions Grice introduces in 'Logic and Conversation'. In what follows I shall make an analysis of Strawson's positions following essentially the ideas put forward by Margáin.[17]

Grice distinguishes between *what is strictly said* when asserting a statement, and what that assertion can *suggest* in a given context to the interlocutor. The statement *implies* everything that is strictly said when it is asserted; in contrast, everything that is suggested but not strictly implied by an assertion is called *'conversational implicature'* of that assertion.

[17] But I will not follow his exposition in detail. I will deal with Strawson's analysis in a more direct fashion (Margain deals with it through the medium of a discussion on Strawson made by Thomas Simpson) and I will add some remarks and the use of a counterexample.

Usually if someone asserts (2), he strongly suggests to the interlocutor that he does not know which of the two persons mentioned was the one who accomplished a certain deed. But that is not *strictly said* by the assertion of (2): if that were the case, (2) would be false provided that the person asserting it knew, indeed, which of the disjuncts is true. Clearly, it is not so. This follows even from the analysis of Strawson: he considers that the truth of one of the disjuncts is a sufficient condition for the truth of (2); naturally, that would not be the case if, when asserting (2), an assertion of ignorance were made also.

One cannot attribute to Strawson an inability to distinguish at all between what is strictly said and what is suggested by an assertion. To refer to the relation between (1) and (2) he employs the verb 'to entail'; to describe the connection between (2) and the suggested ignorance of the speaker he uses the expression 'to carry the implication' (see quotation). The choice of these expressions and his analysis of the truth conditions of 'or' suggest similar distinctions to those of Grice. Nevertheless, Strawson makes a mistake when he attributes a logical role to *implicatures* that only *entailments* can have. His argument against deriving (2) from (1) is that (2) "transmits" something inconsistent with the assertion of (1) (see quotation). Because of the past considerations, that "something" should be classified as an implicature, not an entailment. But then it is absurd to reject the inference: if a supposed conclusion has *entailments* inconsistent with the premise used –this one being consistent– something is wrong with the inference. But if a conclusion can merely *suggest* things incompatible with the assertion of the premise this shows no fault in the argument. A statement Copi has used for other purposes will help us to show this. Let us suppose that the first mate writes in a ship's log the statement, 'Today the captain was sober'. If this note appears *only one day*, this can strongly suggest to the reader that the fact described was a real novelty; in other words, the note can have the implicature that *the captain is seldom sober*. It is an implicature and not an entailment: the first mate can avoid responsibility, alleging that he never *asserted* such a thing about the captain. Let us consider now the statement 'The captain is *always* sober'. It is obvious that it entails the statement written in the logbook (with some implicit premises: that the captain was alive that day, etc.) and in certain contexts it would be natural to infer the latter from this one. Nonetheless, if the reasons alleged by Strawson against Addition in the quotation transcribed were valid, then this inference would have to be rejected too because the conversational implicature that this conclusion has, as we have seen, is incompatible with the premise used. The logical relations between premise, conclusion and the conversational implicature of it, are, in this example, exactly parallel to those of the inferential step criticized by Strawson.[18] The fact that the conclusion of a reasoning can suggest in some contexts[19] something incompatible with the premise, does not prove that such reasoning may have a perfectly intuitive validity.

Far from affecting 'v' and Lewis' argument, Strawson's objections are not even well founded as to the ordinary 'or' (in some *cases* when it resembles 'v'; in *other* usages that I have not discussed here, other considerations of Strawson against Addition are more plausible and are not affected by my observations). Other objections to the rule we are dealing with, posed by Mario Bunge, were discussed in this journal.[20] Bunge began expounding a supposed paradox that would follow from admitting Addition in a note entitled 'The Paradox of Addition and its Dissolution'. Margáin, in his comment 'La Paradoja

[18] In fact, here the supposed logical inconveniences are "stronger": in my example the conversational implicatures of the conclusion are incompatible with *the premise;* in that of Strawson, they only clash with *implicatures* of the premise (the ignorance of the speaker is not incompatible with the authorship of John but, at most, with the speaker's *knowledge* of that authorship or *belief* in it, and that knowledge or belief seems to be a conversational implicature of the assertion of (1)).

[19] Both the conversational implicature of my example and that one of the example offered by Strawson occur only in certain contexts. If somebody attends to the inference from (1) to (2), (2) would lose in that context the suggestion of ignorance that it can have if it is asserted alone (and similarly in my own example).

[20] J. A. Robles reminded me of this discussion, pointing out its relevance to the themes treated in the first version of this paper.

del Dr. Bunge' showed that the derivation of the paradox contained an obvious error. Bunge acknowledged that this was so in a response to Margáin (see Bibliography) but tried at the same time to reformulate his objections to Addition. One of the arguments in his new presentation is based on an ambiguity. Bunge tries to show that with the aid of Addition a paradoxical conditional can be derived from a given premise. Margáin, criticizing Bunge's answer[21] in 'Validez, Inferencia e Implicaturas I' points out (pp. 64-5) that if the conditional in Bunge's conclusion is interpreted as material it is not paradoxical, and if it is interpreted more strongly, then it cannot be derived with the aid of Addition. We have again an illusory paradox. But Bunge's defective argument is accompanied by other considerations about Addition which are more interesting because they seem to be the basis of the intuitive rejection that the rule provokes sometimes. Bunge says:

> Según el principio de adición de la lógica matemática, de p se sigue p ó q, donde p y q son proposiciones que no tienen por qué tener parentesco alguno, sintáctico, semántico o pragmático. (p. 105)

This remark is exact. From '2+2=4' we can infer by Addition '2+2=4 or the moon is green'. Since the disjuncts have nothing to do with each other the conclusion can seem to be senseless, and examples like this often engender in the students a strong intuitive distrust of the legitimacy of the rule. Margáin, in the aforementioned 'Validez...', faces this new evidence with the following consideration:

> No es posible exagerar que la validez de los argumentos en ningún caso garantiza que venga a cuento, sea pertinente o relevante el afirmar la conclusión, si creemos en las premisas. (...) *La validez de los argumentos tiene que ver con la verdad: si las premisas son verdaderas, la conclusión también tendrá que serlo.* Pero la verdad de una oración no ga-

21 In his note 'Comentarios en torno a Bunge, Margáin y la paradoja', Robles criticises other aspects of Bunge's answer, with which we shall not deal here.

> rantiza su pertinencia. No todo lo que es verdadero viene a cuento, y puede ser pertinente algo que después resulta falso (p. 66, my underlining).

This defense implicitly acknowledges that Addition can lead to an irrelevant conclusion but aduces that this shows nothing about non-validity because validity is no guarantee of relevance. I agree with Margáin. Nevertheless his argumentation is insufficient for my purposes: Margáin employs the "official" definition of *validity* (see particularly the underlined text) and this was legitimate in discussing with Bunge because he had not objected to that definition. I cannot do the same. A relevant logician who rejects Addition can accept that the rule is *valid* and that *validity* does not entail relevance *in the official sense of validity* and yet insist that the rule is not legitimate in some intuitive sense. Since my aim is to oppose this kind of contention –and not just that of Bunge– I must try to find intuitive evidence, not directly based on the usual definition of *validity,* in favour of the rule. Let us face the task.

It must be observed first that even though the rule leads sometimes to anti-intuitive conclusions, its rejection (as in the already mentioned system of Parry) would leave groundless, applications of it that enjoy general acceptance. If, for instance, we have before us in mathematics a theorem of the form "$m \leqslant n \supset \ldots m \ldots n \ldots$", given "k=k' " we immediately draw the conclusion "...k...k'...". We have automatically employed one universal instantiation, one Addition and one Modus Ponens. The frequency with which theorems of the aforementioned structure appear in mathematics, make this automatic application of Addition very usual.[22]

Bunge would not object to the preceding paragraph. His

[22] In SADAF Alberto Moretti pointed out to me that these uses of Addition are not indispensable since usually a theorem of the structure described is proved by "cases" showing that "$m = n \supset \ldots m \ldots n \ldots$" and "$m > n \supset \ldots m \ldots n \ldots$" hold, which allows you to make the inference using only instantiation and modus ponens if you take directly as a premise the first case of the theorem. But in this paragraph I have not tried to show that the rule is indispensable but that some of its applications are intuitively unobjectionable.

attitude towards Addition is more balanced than the one exemplified by Parry's system: He acknowledges that it is a useful, even necessary, rule and proposes only to restrict it to cases without problems of irrelevancy (and to do this he formulates a criterion based in the predicates that can be used). He would surely consider legitimate the applications just analysed. But I shall try to show now that any restriction of this kind has *also* the effect of obstructing useful and perfectly intuitive applications of the rule. Any philosopher of science accepts this evident logical generalization: if a theory T consists in the conjunction of several hypotheses $p_1, p_2, \ldots p_n$, then the falsity of any of those hypotheses would imply the falsity of T. This generalization applies to any theory, plausible or not, interesting or not. Let us suppose that T is an extravagant theory composed of the conjunction of two hypotheses, p and q, such that they lack any thematic connection between them, in an intuitive sense. Would the generalization just formulated hold for T? It is totally intuitive to accept that it would. T may be extravagant and with little interest but if one of the hypotheses which form it is false, it is evident that T will also be false. In particular, it will hold that from the falsehood of p the falsehood of T=p&q is derived.

In logical notation: $\sim p \rightarrow \sim(p \;\&\; q)$.

But a *standard* proof of this assertion, which proceeds through derivation of $\sim(p \;\&\; q)$ from $\sim p$, uses Addition:

(1) $\sim p$ (hypothesis)
(2) $\sim p \vee \sim q$ (1, Addition)
(3) $\sim (p \;\&\; q)$ (2, De Morgan)

This example shows that to prove an intuitively undisputable logical relation, it may be useful to apply Addition in a case in which the relevance restrictions between the disjuncts of the conclusion do not hold (recall the hypotheses made with respect to p and q).

A bitter opponent of Addition may react to this argument simply adducing that even though the unrestricted law is *useful* in order to prove the logical relation proviously analysed, it is not *indispensable* to that task (some other primitive rule could be adequate for it too; it could even be adopted as an axiom the very formula '$\sim p \rightarrow \sim(p \ \& \ q)$' which is evident enough). Therefore, we would have found no reasons that carry weight to oppose the intuitive pressure against Addition and it would remain advisable to abandon it (as Parry did) or at least restrict it (as Bunge did). But two considerations worthy of attention can reinforce our last analysis. In the first place, the intuitions against the rule seem to weaken a lot when interesting applications of it are shown –and our example would show that even with irrelevant disjuncts such applications can be found. Secondly, no matter how you prove the obvious theorem $\sim p \rightarrow \sim(p \ \& \ q)$ its acceptance leads to Addition through transformations which are intuitive and do not seem to be objectionable from the point of view of relevance. By Contraposition of the *entailment* and Double Negation, the theorem inmediately yields the law of Simplification. This law permits us to derive Addition through the following transformations:

(1) $(\sim p \ \& \sim q) \rightarrow \sim p$	(Simp.)
(2) $\sim\sim p \rightarrow \sim(\sim p \ \& \sim q)$	(Contraposition of (1))
(3) $p \rightarrow (p \vee q)$	(Double Negation and De Morgan in (2))

But usually the learning of the laws of Double Negation and De Morgan finds no intuitive obstacles "as anyone who has taught elementary logic very well knows".[23] Similarly with Contraposition of '$\rightarrow$', which can also be defended with arguments from relevant logic.[24] We are facing, then, a new

[23] These laws are accepted in the system *E* of A & B (see A & B, pp. 107-9, 156).

[24] Following the intuitive analysis of A & B, '$A \rightarrow B$' holds when the requisite of necessity (if the antecedent is true then the consequent is necessarily so) and the requisite of relevance ('A' and 'B' must have some meaningful connection) are satisfied. Let us analyze the law of Contraposition '$(A \rightarrow B) \rightarrow (\sim B \rightarrow \sim A)$'. Under the hypothesis '$(A \rightarrow B)$' it is easy to see that the consequent satisfies the requisite of necessity (classical analyses had established this already); and it satisfies the requisite of relevance also because if the antecedent holds, 'A' and 'B' must have some meaningful connection and the same will be true of '$\sim B$' and '$\sim A$'. Because of this, Contraposition is adopted in *E* too (see A & B, pp. 107-9).

clash of intuitions. Any of the totally intuitive theorems $\sim p \rightarrow \sim (p \,\&\, q)$ or $(p \,\&\, q) \rightarrow p$ lead, through acceptable intuitive transformations, to a rule that engenders distrust in the layman. What intuitions can one choose? Towards the end of II.1 I hinted at a criterion: to see which ones subsist after paying attention to a systematic argumentation. Well, students usually accept Addition after confronting reasons like those expounded in the preceding paragraphs. They do not cling to their rejection as intelligent students do when faced with the hypothesis that the material conditional is a good translation of the natural language conditional: the reason is, that in favour of Addition there are excellent intuitive arguments and only pragmatic considerations in favour of the usual truth table of the conditional.

Usually this amount of argumentation is not needed to attain acceptance of the rule: it is usually sufficient to show its application. This seems to indicate that the student's doubts have to do with the usefulness of the rule (as I hinted before, it is often thought that adding an arbitrary disjunct will not be an operation with "sense") rather than with the fact that p allows us to "extract" p v q in an intuitive sense (instead, there are really doubts of this sort in the case of the formulas '$A \rightarrow (B \rightarrow B)$' or '$(A \,\&\, \sim A) \rightarrow B$').

3. *Transitivity*

There is a truly unusual way of rejecting Lewis' argument and any other derivation of a "paradox of strict implication". It consists in accepting each step of the argument and the rule that justifies it but rejecting the transitivity of the relation of deducibility (or of *entailment*): if you act like this you can accept each step of a deductive chain and reject that the inference from the first to the last is valid. Smiley, in 'Entailment and Deducibility' gives a definition of entailment (for the formulas of the usual propositional logic) that justifies this strategy. With '$\vdash$' for the relation of *entailment,* he states the definition thus:

> $A_1, \ldots, A_n \vdash B$ if and only if the implication $A_1 \& \ldots \& A_n \vdash B$ is a subtitution instance of a tautology $A'_1 \& \ldots \& A'_n \supset B'$, such that neither $\vdash B'$ nor $\vdash \sim(A'_1 \&,,, \& A'_n)$. [i.e., none of the last two formulas considered is a tautology in the usual sense] (p. 240).

This formulation yields a decidible criterion of *entailment*, improving an idea of von Wright that Strawson had shown to be defective.[25] It is easy to see that the criterion validates each step of Lewis' argument but not the inference of the last formula from the first:[26] this *entailment* is not transitive.

A & B are scandalized by this proposal of Smiley. They comment (p. 154): "Any criterion according to which entailment is not transitive, is *ipso facto* wrong". This is not an altogether fair reaction. A close reading of Smiley's essay shows that he does not pretend to give an analysis of the most common concept of *entailment* through the definition quoted: he presents it as an attempt to elucidate the expression 'obviously entails' (see p. 242) and he then gives other criteria of *entailment* which yield transitive deducibility relations (necessary for "serious logical work") and neutralizes Lewis' argument through more usual channels. If the definition is presented with this aim, the lack of transitivity is not objectionable: obvious deducibility is not transitive; as I pointed out before (end of section II.1) the repeated use of very natural inferential steps may lead to unexpected conclusions. Anyway, independently of the aims of Smiley's proposal, it is important to note that it cannot be used to defend relevant logic's points of view and to attack Lewis'argument: if the purpose is to reform on an intuitive basis the analysis of deducibility, it is not possible to abandon *ad hoc* and without significant arguments a property so intuitively close to *entailment* as transitivity (and I do coincide in this with A & B's opinion).

[25] See von Wright, "The Concept of Entailment", p. 181 and the review by Strawson quoted in the Bibliography.

[26] Smiley analyzes another derivation of '$A \& \sim A \rightarrow B$' different from Lewis' argument but his criterion yields exactly the same results when applied to this.

Summary

It is not possible to exhaust a philosophical discussion; but I believe that the analysis of this section together with those of II.3, gives an idea on how to defend Lewis' argument from the main attacks that can be directed against it. My conclusion is that we can look for considerable intuitive support for each one of the assumptions used in the argument and that no well-founded objection has been presented against them. This conclusion, the pragmatic consideration with which I closed II.3 and the rest of section II, suggest that there are not any theoretical reasons for justifying the abandonment of the classical analysis of deducibility and for its replacement by some relevant logic.

IV. PHILOSOPHICAL *STATUS* AND USEFULNESS OF RELEVANT LOGIC

Have the researches in relevant logic any use? Sections II and III suggest a negative answer. But my own attitude is not so sceptical and I would like to expound some particulars about it here. The analyses of II-III rccommend us to reject relevant logic *as a deviant logic:* it would not be useful *as a substitute* for classical logic.

That relevant logic is deviant, is not as clear as it is with other alternative logics that have been proposed and some remarks on this could be useful. Usually a "rival" logical system differs from the classical one because of "losing" some classical theorems. Formally expressed: in the most common cases, if L_1 is proposed as a rival for classical logic (CL), it will be the case that taking into account the set of formulas that can be made with the vocabulary common to both systems (we can have additional vocabulary in L_1) not every formula of this set that is a theorem of CL is also a theorem of L_1. (It is in this sense that Heyting's intuitionistic logic and Lukasiewicz' trivalent logic differ from CL). But in 'Are Relevant Logics Deviant?' (p. 329) Robert Wolf points out that A & B's system *E* does not differ in this way from CL: every theorem of CL that can be expressed in the vocabulary of E is a theorem of E; E does not reject the classical tautologies that can be expressed in its language.[27] This may suggest that E is a "supplementary" logic rather than a "rival" one. Wolf points out, nevertheless, that in E there is a certain rejection of some formal ingredients of CL: not every valid argument of CL is accepted as such in E, the deductive relations among formulas of the vocabulary shared, are not the same (D.S. is, as we have already seen, an example). But a remark of Susan Haack (in *Philosophy of Logics,* p. 229; in *Deviant Logic* she had not considered relevant logics[28]) complicates the scene. The author says that when the relevant logician asserts that, for instance, 'B' does not follow from 'A' and '$A \supset B$', he does not *contradict* the classical logician because he is using the notion of deducibility *in another fashion* and he agrees that *modus ponens* is valid as validity is usually defined. Is there not, then, any authentic rivalry between relevant and classical logics? Susan Haack suggests there is in a certain meta-systematic level: besides saying that certain classical inferences are not relevant in the sense of relevant logic, logicians like A & B maintain the thesis that their concept of validity is more adequate than the classical one, that this one is defective and should be replaced by the first one. It is exactly here where the rivalry between relevant logic and classical logic lies. For instance, the system E does not constitute by itself a deviant logic (as is to be expected: formal systems do not say anything by themselves); the divergence lies in a theory about the conceptual functions E may accomplish and its relation to classical logic. And it is this divergence that we have found to be illfounded.

These last remarks leave an interesting possibility open: if

27 Wolf also points out that, due to this, E does not fall into any of the formal categories (extended, deviant and quasi-deviant systems) in which Susan Haack places non-*standard* logics (see Wolf, p. 330 and *Deviant Logic,* pp. 4-5). He suggests, nevertheless, a slight modification of the definitions, fully according to the spirit of the classification, which yields that E should be considered a quasi-deviant system.

28 Surely this was due in part to the fact that, even if preceded by several articles, the first volume of *Entailment* by A & B appeared a year later (1975) than *Deviant Logic.*

our objections are not to be understood as being against the formal systems of relevant logic but against a certain interpretation of their conceptual function, then maybe those systems could be useful with another theoretical interpretation. I believe this to be the case. Even if *relevant deduction* is not the only legitimate deduction, it is *a kind* of deduction and it could be of a non trivial interest regarding certain intuitions connected with it. Slips like that of Kleene (II. 4) seem to show that a more strict concept of deducibility could be of use in orden to express some important theoretical ideas. The importance of researches like those that lead to *E* is emphasized also by the fact that until recent years the belief that relevance was an obscure concept, unsuitable for adequate formal treatment, was very wide spread (for a representative quotation see A & B, p. 30). Due to this, the formal characterization made in *E*, some metatheorems that show the adequacy of the system to previous intuitions (like the result on "shared variables" of p. 33 and later extensions), the study of which are the rules responsible for irrelevancies and so forth, have a great logical interest. Used merely as an analysis of *another* kind of deduction, *E* (or other systems of relevant logic) would lose its deviant character and be transformed into a system of "complementary" logic.

It is possible that investigations on relevant logic could also be useful as aids in the development of some deviant logical theories. A & B think that *standard* logic fails also in the analysis of other conditionals different from *entailment*. It is obvious that the 'if. . . then' of everyday language expresses sometimes contingent relations without the nuance of logical implication. The only tool classical logic has, to analyse this contingent conditional is the so-called "material implication" and A & B think this notion to be a very bad tool for the task. With regard to this question, I agree with them: I have hinted before that I cannot find any intuitive defense for the usual truth table for the conditional.[29] A & B believe that the '→'

[29] However I do not follow A & B in their extreme theses on this subject. They think that the material conditional is not even used in mathematics. Because of reasons I will expound somewhere else I believe this to be untenable.

of system *R* (one of the two logical "progenitors" of *E*) could be useful to elucidate contingent conditionals of everyday language (since it has not got an ingredient of necessity but it has relevance which, according to their hypotheses, is an ingredient of the natural conditional); the '$\rightarrow$' of *E* is kept for conditionals which contain the ingredient of necessity. But I would like to mention some considerations that can suggest the usefulness of *E* as an aid in researches about contingent conditionals.

My starting point is an analysis proposed by Faris (*Truth-Functional Logic,* pp. 117 and ff.) of the truth conditions of the everyday language conditional. Faris maintains that the condition *E* that I am going to state is a necessary and sufficient condition for the truth of a conditional of the form 'if p, then q':

Condition E: There is a set S of true propositions such that q can be deduced form S with the addition of p.

I think many examples of everyday language support this sort of "deductive-entimematic" interpretation of the conditional. The interpretations that are made by the scientists of computer analysis of mathematical models, give also a non trivial support. The main conclusion reached by the M.I.T. team that studied the mathematical world model World III in order to elaborate the First Report to the Club of Rome[30] has a conditional structure and was formulated by that team so:

> If the present growth trends in world population, industrialization, pollution, food production, and resource depletion continue unchanged (. . .) the most probable result will be a rather sudden and uncontrollable decline in both population and industrial capacity (p. 23).

[30] Published in the book *The Limits to Growth* by Dennis Meadows and others.

What was the inferential road that lead the team to this result? Basically this: the computer used with the World III program had data, considered to be exact,[31] about the variables mentioned in the antecedent of the quoted conditional: population, industrialization, etc. Let us call S that set of data that the members of the team considered true. To S are "added" new premises which constitute the mathematical expression of the antecedent of the conditional transcribed. From S and the added premises, the computer "deduces" the consequent of the conditional expressed in mathematical language. The team's researchers infer the conditional, reasoning as if they accepted Faris' criterion: they accept it because from the antecedent added to a set of propositions considered to be true, the consequent is deduced. This way of analyzing computer "runs" is not unusual.

Faris' proposal has been discussed by several authors. Susan Haack (*Philosophy of Logics*, p. 36) and A. J. Baker have objected to his analysis and L. J. Russell has came out in defense of it.[32] But my intention here was only to point out an interesting fact. Despite the fact of having intuitive support, condition E leads sometimes to totally counterintuitive results. But what is interesting is that these results are obtained when using deductive resources excluded from the system E to derive q from $S \cup \{p\}$. For instance, using condition E it can be easily proved that, if p is false, then for any q it is true that 'If p then q'. We only need to take $S = \{\sim p\}$. From S and p we can deduce anything, but as we already know, this is only so in classical logic, not in relevant ones. Similarly, condition E allows us to infer that if q is true, then 'if p then q' is also true for any p. Here $S = \{q\}$. With classical logic we can deduce q from premises p and q. But in the system E it is not so: if p is not in fact used to "extract" q, its presence in the set of premises is irrelevant and the inference is rejected (the reader can verify that the counterintuitive result reached in the example discussed in II.1. (ii) was produced by a theory

[31] It seems that only the members of the team believed the data to be reasonably accurate, but this does not hinder the logical analysis.

[32] See articles mentioned in the Bibliography.

of the conditional like that of Faris and an irrelevant inference like the one just analyzed). Finally, condition *E* implies also that from a material conditional 'p ⊃ q' the conditional of everyday language 'if p then q' can be deduced (since nobody doubts the inverse implication, this would amount to proving that the everyday conditional has the same truth conditions as the material conditional). The proof is simple: if 'p ⊃ q' is true, we can have $S = \{p \supset q\}$; then, from $S \cup \{p\}$, q can be inferred by *modus ponens*. But also *modus ponens* is rejected in the system *E*.

These examples suggest an interesting hypothesis: to save the intuitive aspect of condition E and in order to avoid its undesirable consequences it could be useful to use it but restricting the deductive resources to those permitted by system E. This kind of analysis of the conditional is explored in A & B. If the application of it to everyday language yielded better results than the use of '⊃', the system *E* would have permitted the elaboration of an interesting divergent logic of the conditional.

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RESUMEN

En la primera parte de este artículo (Crítica 43) expuse las tesis filosóficas del *Entailment* de Anderson y Belnap (en adelante citado mediante la abreviatura 'A & B'), donde se sostiene que no puede haber *deducción* (en un sentido intuitivo de la palabra) sin *relevancia* entre premisas y conclusión. El razonamiento siguiente (en adelante, 'el argumento de Lewis') parece probar lo contrario:

1.	A & –A	(Premisa)
2.	A	(1, Simplificación)
3.	–A	(1, Simplificación)
4.	A v B	(2, Adición)
5.	B	(3, 4, Silogismo Disyuntivo)

Si este razonamiento se acepta, parece que debemos admitir que una contradicción implica lógicamente enunciados temáticamente "desconectados" de ella y con respecto a los cuales tal contradicción sería irrelevante. Para enfrentar este elemento de juicio en contra de sus tesis, A & B rechazan el argumento de Lewis objetando el uso del Silogismo Disyuntivo. En la parte ya publicada de este artículo, he criticado, a mi vez, tales objeciones. Pero un partidario de las tesis "relevantistas" podría atacar el argumento de Lewis por otras vías, objetando algún otro supuesto lógico utilizado en su justificación. Se usan en el argumento dos reglas más (Simplificación y Adición) y, de manera implícita, una propiedad de la relación de deducibilidad (su transitividad). Todos estos supuestos han sido atacados alguna vez por filósofos simpatizantes de los puntos de vista de la lógica relevante.

En esta segunda parte del artículo se examinan propuestas de diversos autores (principalmente E.J. Nelson, P.F. Strawson, Mario Bunge y T.J. Smiley) tendientes a eliminar o restringir el uso de los supuestos lógicos mencionados. Se concluye en todos los casos, después del análisis crítico correspondiente, que no hay elementos de juicio intuitivos que puedan justificar adecuadamente tal eliminación o restricción de los supuestos lógicos cuestionados. Esto refuerza la argumentación de la primera parte del artículo en contra de los puntos de vista filosóficos de la lógica relevante.

Finalmente, en la última sección del trabajo, se examina el *status* y utilidad de los sistemas formales de lógica relevante. Se explica en qué sentido la lógica relevante es "divergente", se defiende la tesis de que en tanto lógica divergente es indefendible y se sugiere que, con otra interpretación filosófica, los sistemas formales desarrollados por los lógicos

relevantes pueden servir de utilidad para efectuar investigaciones lógicas complementarias de las llevadas a cabo en la lógica clásica (por ejemplo, pueden aplicarse tales sistemas al estudio de *otra* relación de deducibilidad —no necesariamente competitiva de la clásica— y de las propiedades de condicionales contingentes del lenguaje ordinario que no parecen asimilables ni al *entailment* ni al condicional material).

[R.O.]