DEONTIC LOGIC AND THE THEORY OF CONDITIONS

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1. Deontic logic was, in origin, an off-shoot of modal logic. It got its decisive impetus from observations of some obvious analogies between the modal notions of necessity, possibility, and impossibility on the one hand and the deontic or normative ideas of obligation ("ought to"), permission ("may"), and prohibition ("must not") on the other hand. In a broad sense, both groups of concepts can be called *modal;* the members of the first group are sometimes referred to as *alethic,* those of the second group as *deontic* modalities. (A preferable alternative to the term "alethic" is perhaps the term "anankastic".)

Beside analogies and similarities, however, there are also a number of striking dissimilarities between the two types of modalities. Many of the problems which have beset deontic logic since its birth are related to these discrepancies. One difference is the absence in deontic logic of an analogue to the principle "$Np \rightarrow p$" of modal logic.[1] That which necessarily is the case is also as a matter of fact the case; but that which ought to be the case is far from being always actually the case. Another formal difference between modal and deontic logic is that, whereas it is obvious that the tautology necessarily is true ("Nt"), it is not intuitively clear that the tautology also ought to be true ("Ot"). The idea expressed by "Ot" does not seem to make good sense. By contrast, "$\sim O \sim t$" seems not only to make sense, but also

[1] I shall not explain here my use of symbols nor the conventions adopted about brackets. I shall assume that the symbolism will be either familiar to the reader or else self-explanatory.

to be true. This formula says that a contradictory state of affairs is not a state which ought to be the case. This is an analogue to the principle "$\sim N \sim t$" of modal logic which can also be written "Mt". The principle "$\sim N \sim t$" is a weaker form of the principle "$Np \rightarrow p$" in as much as the first follows logically from the second (and principles of ordinary propositional logic, PL), but not *vice versa.*

One can display these analogies, and failures of analogy, in the following table:

	Modal logic (System M)		Deontic logic
A1.	$N(p\&q) \leftrightarrow Np\&Nq$	AI.	$O(p\&q) \leftrightarrow Op\&Oq$
A2.	$Np \rightarrow p \quad \sim N \sim t$		$\sim O \sim t \quad Op \not\rightarrow p$
A3.	Nt		$\cancel{Ot}$

There is a further noteworthy difference between the two logics which attracts attention: In modal logic, the interdefinability of the ideas of necessity and possibility through the schema "M" $=_{df}$ "$\sim N \sim$" provokes no serious objection. But the corresponding schema or equivalence in deontic logic is by no means unproblematic. It seems feasible to admit a "weak" notion of permittedness,[2] according to which something *may* be, if and only if it is *not* the case that the contradictory of this thing *ought* to be. The dual of the formula "$O(p\&q) \leftrightarrow Op\&Oq$", *i.e.* the distribution principle "$P(p\text{v}q) \leftrightarrow Pp\text{v}Pq$", holds good of this notion of permittedness. This, however, does not correspond to the way in which permission is normally thought to be distributive over alternatives. If we are told that we may do this thing or that thing, we normally understand this to mean that we may do the one thing *but also* the other thing. The distribution principle, in other words, would seem to be "$P(p\text{v}q) \leftrightarrow Pp\&Pq$". But

[2] I shall use the words "obligatory", "obligatoriness", "permitted", and "permittedness" with a loose and wide meaning which is rougly equivalent with ordinary uses of "ought" and "may". There is also a stricter use of the words mentioned and of the terms "obligation" and "permission", which belongs typically in legal and moral contexts.

this principle goes with a different idea of permittedness from the one which obeys the interdefinition schema "P" $=_{df}$ "$\sim O \sim$". We can call it a notion of *strong* permission. It is related to possibility (freedom) of *choice* between alternatives.

2. In my paper I shall propose a somewhat novel conception of deontic logic. In the light of this conception the discrepancies between deontic and modal which we metioned in the preceding section are placed in a deeper perspective. A prospect is opened for a solution to many difficulties of a logical and philosophical nature associated with the very idea of a "logic of norms".

This new conception regards deontic logic, not immediately as an *analogue* of modal logic, but as a *fragment* of a more comprehensive logical theory which I shall call the Logic of (Sufficient and Necessary) Conditions. Since, however, this Logic of Conditions — as I conceive of it here — is itself a fragment of modal logic, it will *a fortiori* be true to say the same of deontic logic.

To put it quite briefly, the main contention of this paper is as follows: To say that something ought to be, or ought to be done, is to state that the being or doing of this thing is a necessary condition (requirement) of something else. To say that something may be, or may be done, again has two different, typical meanings. Either it is simply a *denial* of the statement that the contradictory of this thing ought to be (done), *i.e.* is a necessary requirement of something else. Or it is an *affirmation* to the effect that the being or doing of the thing in question is a sufficient condition (guarantee) of something else. When a statement of permittedness has the first meaning, it can also be rendered as a "need not"-statement. When it has the second meaning it is frequently couched as a "can"-statement. I shall refer to the two meanings as the *weak* and the *strong* (meaning of) "may" respectively.

In calling the proposed conception of deontic logic "somewhat novel", I have in mind the existence of an earlier attempt in the same direction. This is A. R. Anderson's well-known reduction of deontic to alethic modal logic.[3] Substantially, the present paper is a further development and extension into new dimensions of ideas originally put forward by Anderson.

3. A satisfactory logical theory of conditions is still very much of a *desideratum*. We cannot attempt to satisfy it here. But we must agree about some of the features of this theory before we can proceed to considering its connexion with deontic logic.

A theory of conditions can conveniently be built in stages. On the lowest stage, the terms of the conditionship-relation are propositions (or some "proposition-like" entities). These are not further analyzed, but treated as "wholes". The laws of "classical", two-valued propositional logic (PL) are accepted for them. What else is needed? At least, I think, the modal ideas of necessity, possibility, *etc.* and some minimum assumptions about their logic. A plausible minimum seems to be to accept the system of modal logic known as System M (or T). Its axioms were listed above, in Sect. *1.* The system is the common core of most modal logics which are known and studied; it embodies the unproblematic, more or less universally "received" principles of modality.[4]

We tentatively suggest the following definition of the notion of a necessary condition, strictly speaking of the phrase "the truth of the proposition that p is a necessary condition of the truth of the proposition that q":

"$Nc\,(p,q)$" $=_{\mathrm{df}}$ "$N(q \rightarrow p)$".

Thereupon we suggest the following definition of the

[3] "A Reduction of Deontic Logic to Alethic Modal Logic", *Mind 67*, 1958.

[4] Another treatment of the elements of a logic of conditions is found in my book *A Treatise on Induction and Probability* (1951). That treatment is framed, not in modal propositional logic, but in the monadic lower functional calculus (theory of properties and quantifiers).

phrase "it ought to be the case that p":

"Op" $=_{df}$ "$Nc\ (p,I)$".

That something ought to be the case, or is "obligatory" in a loose and wide sense of that term, thus means according to our suggestion that the thing in question is a necessary condition of a certain thing (proposition, state of affairs) "I". This is a propositional constant, not a variable. What this "I" is, its content, we leave for the time being open.

On the above basis it is easy to prove "$O(p\&q) \leftrightarrow Op\&Oq$", or the "received" distribution principle for the deontic O-operator.

"$Op \rightarrow p$" is *not* a theorem of our theory (of conditions). This is as we want to have it: It must not follow logically from the fact that something *ought to be* that this thing *is.*

"$\sim O \sim t$" is *not* a theorem either. This conflicts with the traditional conception of deontic logic. A way out of the conflict is opened by the observation that the only thing of which a contradictory state of affairs can be a necessary condition is — another impossible state. "$\sim O \sim t$" would be provable, if we modified our definition above of the notion of necessary condition by stipulating that that of which something is a necessary condition must not itself be impossible.

With or without this additional stipulation, however, "Ot" is a theorem. This too is at variance with "traditional" conceptions. As noted in Sect. *1*, "Ot" is not particularly wanted as a theorem of deontic logic; we may even wish to reject it. We can get rid of the unwanted result by stipulating that that which is necessary condition of something must not itself be necessary.

The two stipulations — the one which secures theoremhood for "$\sim O \sim t$" and the one which excludes "Ot" from the the status of a theorem — are embodied in the following modified definition:

"$Nc\ (p,q)$" $=_{df}$ "$N(q \rightarrow p)\&M{\sim}p\&Mq$".

Let the symbol "C" signify contingency. If we accept the modified definition of a necessary condition, we can prove the theorem "$Nc(p,q) \rightarrow Cp\&Cq$". From our definition of "ought" we then also prove "$Op \rightarrow Cp$" and, since contingency entails possibility, "$Op \rightarrow Mp$". The last may be regarded as a version of the principle, commonly associated with the name of Kant, that "Ought implies (entails) Can".

I shall call the addition "$M{\sim}p\&Mq$" to the definition of Nc a *contingency-clause.*

Someone may wish to object to the new definition that it is an *ad hoc* modification for the sake solely of accommodating deontic logic within a theory of conditions. My rejoinder will consist of two parts:

First, I do not think the objection a fair one. Quite apart from considerations relating to deontic logic and normative concepts, the suggested modification seems to me reasonable. But it challenges the serious problem of how to deal with relations of conditionship between non-contingent, *i.e.* necessary or impossible, propositions. Since this problem is not relevant to deontic logic, we need not discuss it at length here. Let it only be said that I think its solution has to be given in terms of higher order (iterated) modalities. A necessary condition of a necessary proposition, I submit, is not a necessary condition of the truth of that proposition, but of its necessity; and a necessary condition of an impossible proposition is not a necessary condition of the falsehood of that proposition, but of its impossibility.

Secondly, instead of modifying the definition of a necessary condition we could correspondingly modify the definition of the O-operator by adding to it the requirement that the states "p" and "I" should be contingent. This would seem entirely unobjectionable and would lead to essentially the same results as far as deontic logic is concerned. But, as indicated, I should favour the more daring modification proposed above.

Let it be observed in passing that "$Op \rightarrow O(p \vee q)$" is *not* a theorem of the deontic logic we are building. This is the famous Ross's Paradox formula. The fact that it is not a theorem does not mean, however, that the notorious troubles caused by this paradox, and its variations, have been completely overcome. For, under a conditional clause to the effect that the disjunction "pvq" is not necessary, *i.e.* the conjunction "$\sim p\&\sim q$" is possible, the formula becomes derivable. It is easy to find examples of propositions which satisfy this clause, — *e.g.* the stock example "if it ought to be the case that this letter is mailed, then it also ought to be case that this letter is mailed or burnt" will obviously satisfy it.

The fact that Ross's Paradox formula is not a theorem reflects a restriction on the validity of the distribution principle for the *O-operator*. It does not follow logically from the fact that the conjunction of two states is contingent that each one of the states individually is contingent; one of them may also be necessary. The implication "$O(p\&q) \rightarrow Op\&Oq$" therefore holds only subject to the conditional clause "$M\sim p\&M\sim q$". Similarly, it does not follow logically from the fact that two states individually are contingent that their conjunction is contingent; the states may exclude or contradict each other, in which case their conjunction is impossible. The implication "$Op\&Oq \rightarrow O(p\&q)$" therefore must be subject to the conditional clause "$\mathrm{M}(p\&q)$". We can contract the two clauses into one and get the theorem:

"$M\sim p\&M\sim q\&M(p\&q) \rightarrow (O(p\&q) \leftrightarrow Op\&Oq)$".

4. I now proceed to the sufficient condition aspect of deontic logic.

The definition of the notion of a sufficient condition which corresponds to the definition which was first suggested for the notion of a necessary condition is as follows:

"$Sc(p,q)$" $=_{\mathrm{df}}$ "$N(p\rightarrow q)$".

A sense of the phrase "it may be the case that p" is then defined as follows:

"Pp $=_{\mathrm{df}}$ "$Sc(p,I)$".

According to this suggestion, that something may be the case, or is "permitted" in a loose and wide sense of this term, means that the thing in question is a sufficient condition or guarantee of a certain state "I". What this "I" is will for the time being be left open.

The symbol "P" will henceforth be used *only* for the above sense of "may", and not for the sense which is also expressed by "$\sim O \sim$".

From PL and M and the suggested definitions of "Sc" and "P" we easily prove the distribution law "$P(p\vee q) \leftrightarrow Pp\&Pq$" or that it may be the case that p or q, if and only if, it may be the case that p but also may be the case that q.

On the basis of considerations of symmetry alone, but also for other reasons, it may be thought that, if the relation of necessary conditionship is restricted by definition to contingent propositions only, the relation of sufficient conditionship should be similarly restricted. We achieve this restriction through the following modification of the definition:

"$Sc(p,q)$" $=_{\mathrm{df}}$ "$N(p \rightarrow q) \& Mp \& M \sim q$".

Both under the unmodified and the modified versions of the definitions of "Nc" and "Sc" these equivalences hold true: "$Sc(p,q) \leftrightarrow Nc\ (q,p)$" and "$Sc(p,q) \leftrightarrow Sc(\sim q, \sim p)$" and "$Nc(p,q) \leftrightarrow Nc\ (\sim q, \sim p)$".

Accepting the modified definition of "Sc", we have to note a restriction on the validity of the P-distribution principle, analogous to the restriction we noted in Sect. *3* for the O-distribution principle. It does not follow logically from the fact that the disjunction of two states is contingent that each one of the states individually is contingent; one of them may be impossible. The implication "$P(p\vee q) \rightarrow Pp\&Pq$" therefore requires a conditional clause "$Mp\&Mq$". Similarly, it does not follow logically from the fact that two states individually are contingent, that their disjunction is contingent; if the states are contradictories of one another, their disjunction is necessary. The implication "$Pp\&Pq \rightarrow P(p\vee q)$" therefore requires the conditional clause "$M(\sim p \& \sim q)$". Contracting the two clauses we have the theorem:

"$Mp\&Mq\&M(\sim p\&\sim q) \rightarrow (P(p\vee q) \leftrightarrow Pp\&Pq)$".

Assume that "p" is something which, in the strong sense, may be and that "q" is something which ought to be. Then we have, according to our definitions, "$N(p \rightarrow I) \& N(I \rightarrow q)$". This entails "$N(p \rightarrow q)$" which entails "$p \rightarrow q$". Thus if something which, in the strong sense, *may* be the case actually *is* the case, then everything which *ought to* be the case *is* the case, too. Of that which is in the strong sense permitted one can, in other words, avail oneself only on condition that none of one's obligations is thereby violated. This is a reflexion, in deontic terms, of the general principle for conditions which says that a sufficient condition of something can be realized only provided that all the necessary requirements for the occurrence of the thing in question are satisfied as well. To say of something that it is in the strong sense permitted without including in the description mention of all dutybound things is therefore an *elliptic* mode of speach. Similarly, to say that something is a sufficient condition of something else without mention of the necessary conditions is elliptic, too. This elliptic mode of expression is, however, quite commonly used.

5. Accepting the modified definitions of the notions of necessary and of sufficient condition, it is easily shown that "$Pp \rightarrow \sim O \sim p$" is a theorem. If something may be, then it is not the case that its contradictory ought to be. Now, as we have already noted, there is a weak sense of "may" which means simply that whatever is such that its contradictory is *not required* (for a certain thing) "may" be. This is the notion of permittedness which figures in the traditional calculi of deontic logic. The fact that it is entailed by our new P-notion justifies us in calling the latter the *strong* and the former the *weak* notion of "may" (or of permittedness).

"$Op \rightarrow \sim O \sim p$" is another theorem. If something ought to be, then it may be — in the weak sense of "may". But may it also be in the strong sense? The answer is negative.

"$\sim O \sim p \rightarrow Pp$" is not a theorem. This observation is related to a problem which has been very much discussed in the theory of jurisprudence. The problem is whether anything which is not prohibited ("$\sim O \sim p$") is *ipso facto* permitted. The principles of legal philosophy couched in the Latin words *nullum crimen sine lege* and *nulla poena sine lege* seem to favour an affirmative answer. Accepting this answer for the whole of a given legal order would mean that this order is *closed,* has no "gaps" or "lacunae" in it. Some legal philosophers have accepted this and some have even thought that a legal order is of necessity closed. This appears to be the position, *e.g.*, of Hans Kelsen. But most legal theorists seem to think that the closed or open nature of a legal order is a matter of contingent fact. A difficulty for those, who follow this line of thought, has been to show *why* it is that — speaking in the terms of deontic logic — "Pp" does not follow logically from "$\sim O \sim p$". The clue to a solution of the problem lies in the recognition of two different concepts of permission.

Assume that the conjunction of *all* the necessary conditions of some state of affairs is a sufficient condition of this state. Then I shall call the state of affairs in question *determined.* For from this assumption it can be concluded that, whenever the state obtains, some or other of its sufficient conditions also obtains. To assume that *all* states of affairs are, in this sense, determined would be tantamount to assuming that no state can obtain (occur) in the absence of a sufficient condition for it (its occurrence). This captures, I think, an important aspect of what traditionally goes under name of Universal Determinism. (The idea can also be relativized to some *class* of states.)

The question of universal determinism does not concern us here. But let us see what follows if we assume that the particular state "I", which has been used in our definitions of "ought" and "may", is determined. This assumption means that, if everything that ought to be the case actually, on some given occasion, is the case, then whatever else there is (happens to be) may be the case. Or, speaking in juristic terms: if all obligations are satisfied, then whatever else is done (or omitted) is also allowed. The "system" of obligations and permissions defined in terms of this "I" is *closed.* The idea of closedness of legal (and other normative) orders is thus a special case of a more general idea of determinism.

An example will be given to illustrate the above. Assume that "I" is determined and that the conjunction of all its necessary conditions is equivalent with "$p\&q$". Then we have "$O(p\&q)$" and "$P(p\&q)$". Consider now a state "r" which is, as I shall say, co-contingent with "$p\&q$". By this I mean that "$M(p\&q\&r)$" and "$M(p\&q\&\sim r)$" are both of them true. Then we can derive "$P(p\&q\&r)$" and "$P(p\&q\&\sim r$"). Of course, neither "r" nor its contradictory "$\sim r$" is *by itself* a sufficient condition of "I". "Pr" and "$P\sim r$" are not derivable. But we can introduce here a new piece of terminology and say that, if a state of affairs is co-contingent with the conjunction of all the necessary conditions of a determined state "I", then this state and also its contradictory state are, *in the frame or setting of the necessary conditions,* sufficient conditions of "I" and in this sense also strongly permitted. (Cf. what was said in Sect. *4* on the elliptic use of "permitted" and "sufficient condition".) Since, moreover, neither the state in question nor its contradictory is itself a necessary condition of "I" — this would conflict with the assumption of co-contingency — both states are in the *weak* sense permitted. "$\sim O\sim r$" and "$\sim Or$" are derivable.

6. A notion which has caused students of deontic logic considerable trouble is that of commitment. It is related to the notion of a conditional obligation (norm).

A "formalization" of commitment which has been suggested is "$O(p \rightarrow q)$". The formula can be read "it ought to be the case that, if it is the case that p then it is also the case that q". If it is in the power of an agent to produce the two states ("at will"), then this agent will, by producing the first state, commit himself to producing the second state as well. This means: unless he now produces the second state, he will have done something forbidden.

The suggested "formalization" is open to the objection that, if the first state is, in itself, forbidden and the second is not in itself, necessary, then doing the first commits one to doing the second. From "$O{\sim}p\&M{\sim}q$" follows "$O(p \rightarrow q)$". This is, in fact, simply a version of Ross's Paradox.

Another suggestion as to how to "formalize" commitment, is "$p \rightarrow Oq$". This evades the paradox that doing the forbidden "commits" one to doing anything whatever (which is not in itself necessary). But then we run into the new "paradox" that relative to that which *is not* (whatever it be) any other thing ought to be.

When deontic logic is placed in the setting of a theory of conditions, these difficulties can be overcome. Consider first the following point about necessary conditions:

A necessary condition can have disjunctive form. That "p or q" be the case can be a necessary condition of "r", say. Now assume that "p" is *not* the case. Then, relative to this, "q" so to speak "becomes" necessary, if "r" is to occur at all. How shall this be expressed? "$Nc\ (pvq,r)$" by itself cannot express it. That "q" is a necessary condition of "r", if "p" is not there, *entails* (presupposes, requires) that "pvq" is a necessary condition of "r", but is a stronger statement. "${\sim}p \rightarrow Nc(q,r)$" will not do either. For it says that, when "p" is not, then *any* state is a necessary condition of "r" — and this is certainly not intended. Nor does this expression entail "$Nc(pvq,r)$".

Consider, however, "$Nc(Nc(q,r),{\sim}p)$" or, which means the same "$Sc({\sim}p,Nc(q,r))$". The last formula says that the absence of the state "p" is a sufficient condition for the state "q" to be a necessary condition of the state "r". If we make the assumption that it is possible for both the two states, "p" and "q", to be absent, *i.e.* if we assume that their disjunction is not in itself a necessity, then the above second order expression entails that the disjunction of the state "p" and the state "q" is a necessary condition of the state "r" — though not *vice versa.* We have, in other words, "$Sc(\sim p, Nc(q,r))\&\sim N(pvq) \rightarrow Nc(pvq,r)$".

Now substitute "I" for "r". Then we get from the last formula "$Sc(\sim p, Oq)\&\sim N(pvq) \rightarrow O(pvq)$". The antecedent, I suggest, is an expression for commitment (relative, conditional obligation). I shall define a new deontic operator Q as follows:

"$Q(q/p)$" $=_{df}$ "$Sc(p,Oq)\&\sim N(p \rightarrow q)$".

"$Q(q/p)$" can be read "it ought to be the case that q, *given* that p". Assuming that it is in the agent's power to produce "p", then by producing this he "becomes" obligated to produce also "q", unless "q" is something which is not of necessity there as soon as "p" is there. (For in this last case, it "makes no sense" to speak of an "obligation".)

"$Q(q/p)$" entails "$O(p \rightarrow q)$" but not *vice versa.* "$O\sim p$" does not entail "$Q(q/p)$". Nor does "Oq" do this.

Of particular interest is the case when "$O\sim p$" actually obtains with "$Q(q/p)$". Then the second expression tells us what the agent ought to do, when he has done something he was, in fact, forbidden to do. This sort of commitment or conditional obligation is of the kind for which Professor Chisholm has coined the name Contrary-to-Duty Imperative.[5]

The above ideas seem to me to lead to a satisfactory account of the notion of commitment, and of conditional norms generally. Some problems of old standing in deontic logic now acquire a natural solution.

7. The incorporation of deontic logic into a modal logic of conditions also opens new prospects for dealing with problems relating to the iteration of deontic operators. In "tra-

[5] R. M. Chisholm, "Contrary-to-Duty Imperatives and Deontic Logic" *Analysis 24,* 1963.

ditional" deontic logic these problems are notoriously obscure.

That deontic operators can become iterated is no more of a "mystery" than that conditions can be of higher order. The concept of commitment, and of a conditional norm generally, is — as we have just seen — a second order condition concept. In it is involved the idea that the fact that something is a necessary condition of something else is itself a necessary condition of something.

If we let these things of which something is a necessary condition be identical with the state of affairs "I" which figures in our definitions of the deontic operators, we obtain higher order deontic expressions such as "OOp", "OPp", "POp", "PPp". Thus, for example, "OOp" when explicated in terms of conditions means "$Nc(Nc(p,I),I)$". When further explicated in modal terms we get from this the expression "$N(I \rightarrow N(I \rightarrow p)\&MI\&M{\sim}p)\&MI\&M{\sim}\ (N(I \rightarrow p)\&MI\&M{\sim}p)$". If we ignore, or take for granted, the contingency-conditions attached to the antecedents and consequents of the two strict implications involved, we can simplify the formula to "$N(I \rightarrow N(I \rightarrow p))$".

In the light of this interpretation of higher order deontic expressions one can profitably examine some formulae, whose status as logical truths about the deontic modalities has been defended by some and disputed by others. I shall consider one example. This is a formula which I shall label Prior's Formula.[6]

It is clear that "$Op \rightarrow p$" cannot be a truth of logic. (Cf. Sect. 1.) But what about "$O(Op \rightarrow p)$"? In words this formula says that it ought to be the case that that which ought to be the case actually also is the case. This sounds reasonable enough. Let us see what it amounts to when the formula is translated into the terminology of conditions and modalities.

[6] It was introduced into the literature by A.N. Prior in his *Formal Logic* (1955).

As a statement of conditionship the formula means "$Nc((Nc(p,I) \rightarrow p),I)$". If we write it out in modal terms we get "$N(I \rightarrow (N(I \rightarrow p)\&MI\&M(N(I \rightarrow p)\&MI\&M{\sim}p\&{\sim}p)$".

Let us first ignore the contingency-clauses and "pull out" the strict implication part of the formula. Then we get "$N(I \rightarrow (N(I \rightarrow p) \rightarrow p))$". This is easily shown to be a theorem of modal logic (M). Therefore the conditional statement which says that, if the contingency-clauses are satisfied, then Prior's formula holds good, is a theorem of modal logic, too.

We now turn attention to the contingency-clauses. The third member of the main conjunction in the modal expansion of Prior's formula can be abbreviated to "$M(Op\&{\sim}p)$". It says that it is possible that the state of affairs "p" *ought to be* but nevertheless *is not.*

But is this clause needed at all? Is it not trivially satisfied by virtue of the fact that "$Op \rightarrow p$" is not a theorem? The answer to the last question is negative. The mere fact that "$Op \rightarrow p$" and therewith also "$N(Op \rightarrow p)$" is not a theorem does not make the negation of "$N(Op \rightarrow p)$" which is equivalent to $M(Op\&{\sim}p)$" a truth of modal logic. Whether the clause upon which Prior's formula is conditional is satisfied or not is therefore a matter of contingent truth and not of logical necessity.

What the "if-clause" which is needed to warrant the truth of Prior's formula does, interestingly, is to draw attention to the fact that, although it is inherent in the notion of an obligation that the obligatory is something *in itself* contingent and in this sense neglectable, it may yet *as obligatory* be impossible to neglect. "$Op \rightarrow M{\sim}p$" is a theorem of deontic (modal) logic. But "$Op\&M(Op\&{\sim}p)$" can be true or false, and so can "$Op\&N(Op \rightarrow p)$". When "$Op\&M(Op\&{\sim}p)$" is true of a state "p", I shall say that the fact that "p" ought to be is a *neglectable obligation.* When again "$Op\&N(Op \rightarrow p)$" is true of a state I shall say that the fact that this state ought to be is a *non-neglectable obligation.*

I conclude this section with the conjeture and suggestion that the distinction which we have discovered between two types of obligation can be interestingly related to things familiar from traditional ethical theory. (Over-riding obligations, *prima facie* obligations, *etc.*)

8. In our definitions of the deontic notions we have so far employed an unspecified propositional constant "*I*". Something will now have to be said about its possible content.

One suggestion could be that no specification at all of the content of "*I*" is needed for a definition of "ought". According to this view, to say that something ought to be or ought to be done is to say that the being or doing of this thing is a necessary condition of a certain other thing which is taken for granted or presupposed in the context. An "ought"-statement is typically an *elliptic* statement of a necessary requirement. The same holds, *mutatis mutandis,* for the two types of "may" statement. (This elliptic use, omitting reference to a specified "*I*", must not be confused with the elliptic use mentioned in Sect. *4* and *5* above, when reference to a frame of necessary conditions is suppressed but tacitly presupposed.)

This suggestion seems to me, on the whole, acceptable. If we accept it, then we are always, when confronted with an "ought", entitled to raise the question "Why?", *i.e.* to ask for the thing for which this or that is alleged to be a necessary requirement. There may exist a tendency, particularly in so-called moral contexts, to forget about the elliptic character of the "ought", and to accord to it an absoluteness which intrinsically it cannot have. It is an important aspect of the task of the moral critic to challenge accepted "oughts" by raising for them the question "Why?" or "What for?"

Even if we accept that the "*I*" in our definition of "ought" can be any state whatever, which is capable of having necessary conditions, there are some *types* of case which are worthy of special attention. The distinctions between the types have partly to do with the content of "*I*", and partly also with the kind of necessity involved in the contemplated conditionship-relation.

One important type of case is when "*p*" is thought to be a logically (conceptually) necessary requirement of "*I*". For example: A good fountain-pen, we say, ought not to leak. Or: In order to be legally valid a marriage ought to be concluded in the presence of two witnesses. A leaking fountain-pen does not count or qualify as a good one — and a marriage at which two witnesses have not been officially recorded does not count or qualify as valid. But the standards of legal validity may be different under different legal systems; and also standards of goodness in fountain-pens can vary depending upon preferences.

Another characteristic type of case is when the state "*I*" is the aim or end of some agent for the attainment of which the being or doing of "*p*" is thought to be *causally* necessary. The thing which ought to be or be done is then required as a necessary means to the given end. An example would be when I say: "in order to get there in time I ought to hurry".

Stipulations or rules conforming to the first pattern of "ought" are or resemble *definitions*. Rules or norms in accordance with the second pattern of "ought" can be called *technical norms* or alternatively *practical necessities*.

The "may", both in its weak and strong sense, can also be of one of the two types which we just described. If I say that a good fountain-pen *need not* be of gold but *may* be of some material other than gold, I mean that this particular requirement concerning the material of which it should be made is not built into the (my) concept of a good fountain-pen. This is the weak "may". Similarly in the case when I say that in order to get to a destination in time I need not take a tram, but may also use some other means of transportation. Assume, however, I said that in order to get there in time, I *may* take a bus. This would normally mean that, if I go there by bus I shall arrive in time, but that there perhaps are other means beside this to secure the same end of my action. I may, *e.g.*, also take the subway, I can choose between it and the bus. This is the strong "may".

The problem of the content of "*I*" is of particular interest to philosophy, when the "ought" (and the two "may") is a *legal* or a *moral* "ought" ("may").

I would submit for consideration that for an important type of legal "ought" — the "ought of legal obligation" as I shall call it — "*I*" is a state of affairs which can be characterized as *immunity to punishment* (a punitive reaction on the part of a legal machinery). The actions which it is our legal duty or obligation to do are those which are required of us if we are to be immune to legal punishment. They are *necessary* to ensure immunity, but whether they are also *sufficient* depends upon whether the legal order, or part of legal order, under consideration is closed or open. (Cf. above Sect. 5.)

But what is immunity? This is itself a problematic notion and can be understood in several ways. One could suggest this: Immunity means that, unless "*p*" is (done), punishment will follow. But this would hardly be an interesting notion of immunity. For "*p*" may be neglected and yet no punishment follow, either because the criminal is not caught or, although caught, acquitted because not proved guilty. To be immune to punishment for neglect of "*p*" means rather that one *cannot* be legally punished for having neglected "*p*". And the contradictory of immunity, which can also be called *liability* to punishment, means that one *can* be thus punished. But what do "can" and "cannot" mean here? They are modal notions and on that ground alone related to "ought" and "may".

In the analysis of the notions of immunity and liability to punishment, deontic notions may thus crop up anew. This would not entail circularity or show that immunity and liability cannot be interestingly used for defining the notions of legal obligation and permission. But it would show that in the structure of a legal order other types of "ought" and "may" than (legal) obligations and permissions are involved — and it would challenge further analysis of how the various types of norm which build up this order are related and intertwined.

9. Condition-statements, for example to the effect that something is a necessary or is a sufficient condition of something else, are *genuine statements*. By this I mean that it makes sense to attribute truth-value to them, to speak of them as being true or false. It does not follow that they are "naturalistic", if by that is meant that the grounds on which they are pronounced true or false are experiential (empirical). When the condition-statement is about the necessary or sufficient means to some end, its truth-grounds are — normally at least — experiential and the statement "naturalistic". When, on the other hand, the statement is about the logically necessary requirements for falling under a concept (qualifying as a such-and-such), the truth-grounds are not experiential — and there need not exist any truth-*grounds* for the statement at all.

"Ought" and "may" have an important connexion with *norms*. Norms are not ordinarily called "statements" or "propositions". Whether truth-value can be attributed to norms is a matter of controversy. Many philosophers and logicians have thought that norms are essentially void of truth-value, "outside the realm of truth and falsehood", belong to "practical" as distinct from "theoretical" discourse. And some of these philosophers have therefore thought that norms are essentially a-logical, that there can be no such thing as a "logic of norms" or deontic logic.

It is important that the "theoretical" character of condition-statements should be reconciled with the "a-theoretical" aspect of norms — the "ought" and "may" which express necessary or sufficient conditions with the "ought" and "may" of genuine norms. Such reconciliation is, I think, fully possible — and to effect it is a philosophically relevant task. A satisfactory accomplishment of it would remove much confusion which has of old prevailed in legal and moral philosophy.

Why is it that to say that something ought to (or may) be the case or be done so often has the appearance of not being a genuine statement, to which a truth-value can be significantly attached?

There are several reasons why this is so. — *One* is, I think, to be sought in the *elliptic* character of many "ought" — and "may"-statements. If someone says "this ought to be" and another "that ought to be" and the "this" and the "that" exclude each other and there is a dispute, we cannot begin trying to settle it until we have first stated *for what* the "this" and the "that" are thought to be required. If they are required for different things, there can be no dispute between the two "oughts" (but perhaps another dispute about which of these two things ought to be pursued). If they are required for the same thing (the same "*I*"), one of the disputants may be right and the other wrong or both may be wrong. And now "right" and "wrong" is a matter of truth and falsehood.

Even when the elliptic character of a given "ought" (or "may") is recognized in principle it may be difficult or even impossible to specify in practice the "*I*" relative to which something ought to or may be. Perhaps we only have some dim conception of its nature. (We had been taught to think the matter was clear without questioning. This is what all too often happens in an authoritarian type of society.) Then we may not come to a "grip" with the question of truth in connexion with the "ought" ("may") and the norm takes on an "alogical" appearance.

A characteristic use of "ought" (not so much it would seem of "may") is to *evince an evaluation.* Thus in saying "a good fountain-pen ought not to leak" or simply "a fountain-pen ought not to leak", may be evincing a standard of goodness in fountain-pens. "A leaking fountain-pen, according to my conception of the matter, simply *is* not a good one." To adopt this standard is to *make* the property of not leaking a necessary requirement of goodness in fountain-pens.

Another characteristic use of "ought" (and to some extent also of the strong "may") is for enforcing patterns of behaviour (conduct). This is, in a paramount sense, the *normative* use of the words "ought" and "may". *Imperatives* (sentences in the imperative mood) are also used for the same purpose. It would be a confusion to say that "ought"-*statements* are imperatives. Imperatives are not statements and imperatives, as has so often been pointed out, are not true or false. But "ought"-sentences are commonly and characteristically *used as imperatives, viz.* used for the purpose of urging (making) people to behave in a certain way.

It is futile to dispute whether "open the window" and "you ought to open the window" mean (as it were "intrinsically mean") the same or not. Sometimes someone addresses another person with the words "you ought to open the window", when he could just as well have said "open the window". But perhaps he will, when he says the former, more often than not have "at the back of his mind" an idea of the thing for which compliance with the order is a necessary requirement, — *e.g.* that ventilation of the room is badly needed. And one can be pedantic and say that "ought" is *properly used* only when there is an answer to the question "Why?", whereas use of the imperative can also be appropriately made when this question is out of place.

The use of "ought" for giving commands and orders and the use of "may" for giving permissions is a species of *performative* use of language. Language, when used performatively, can rightly be said to be "outside the categories of truth and falsehood". But the same sentence which has a characteristic performative use can also have a characteristic descriptive use. And the two types of use may *fuse.*

10. Finally, some peculiarities on *linguistic usage* will be noted.

When we say " '*p*' *and* '*q*' *and* . . . is a *necessary* condition of —" we intimate that "*p*" and "*q*" and . . . are *all* of them, individually, necessary conditions of —.

When we say " '*p*' *or* '*q*' *or* . . . is a *necessary* condition of —", we intimate that "*p*" and "*q*" and . . . are *none* of them individually, nor *some-but-not-all* of them disjunctively, a necessary condition of —.

When we say " '*p*' *and* '*q*' *and* . . . is a sufficient condition of —", we intimate that "*p*" and "*q*" and . . . are *none* of them individually, nor *some-but-not-all* of them conjunctively, a sufficient condition of —.

When we say " '*p*' *or* '*q*' *or* . . . is a *sufficient* condition of —", we intimate that "*p*" and "*q*" and . . . are *all* of them, individually, sufficient conditions of —.

The second and the third case, *i.e.* "or" in connexion with necessary and "and" with sufficient conditions, bear on anomalies of the type of Ross's Paradox.

The "and" and "or" in these locutions are *not* identical with the truth-functional notions of conjunction and disjunction.

Compare "ought to", "should", "must", "has to". Is not "must" stronger than "ought to", more expressive of a necessary requirement (condition) than "ought to"? So it is, but I do not think one can make hard and fast distinctions here. Even necessity can possess degrees of rigour or laxity. Sometimes higher order modal concepts can be used for expressing this. That which is necessarily necessary may in an interesting sense be said to be "more necessary" than that which is only contingently necessary. If there existis any way at all of capturing the distinction between "ought to" and "must" by means of logic, this would have to happen within a theory of conditionship-relations of different orders. (Cf. what was said in Sect. *7* about neglectable and non-neglectable duties.)

RESUMEN

La lógica deóntica comenzó siendo una rama de la lógica modal, con la que guarda importantes semejanzas. Al lado de esas analogías, sin embargo, se encuentran discrepancias fundamentales, como puede verse en el siguiente cuadro:

Lógica modal (Sistema M)	Lógica deóntica
A1. $N(p\&q)\leftrightarrow Np\&Nq$	A1. $O(p\&q)\leftrightarrow Op\&Oq$
A2. $Np\rightarrow p \quad \sim N\sim t$	$\sim O\sim t \quad Op\not\rightarrow p$
A3. Nt	$\cancel{Ot}$

En este artículo se presenta una nueva concepción de la lógica deóntica que coloca las anteriores semejanzas y diferencias en una perspectiva más profunda, y permite solucionar muchas dificultades lógicas y filosóficas asociadas con la idea de una "lógica de normas". La lógica deóntica pasa de un simple análogo de la modal, a un fragmento de la nueva lógica de las condiciones necesarias y suficientes. Los conceptos fundamentales de la lógica deóntica serán, pues, definidos en términos de condiciones.

Aunque no se ha dado, todavía, una teoría satisfactoria de las condiciones, podemos estar de acuerdo en sus rasgos fundamentales. Tenemos, primero, a las proposiciones, unidades no analizadas que son los términos de la relación condicional. Se aceptan las leyes de la lógica proposicional bivalente clásica (PL). Las ideas modales de necesidad, posibilidad, etc., son indispensables. También conviene aceptar el sistema M de lógica modal, pues recoge principios "recibidos" y no problemáticos.

Se sugiere la siguiente definición de la "verdad de la proposición que p es condición necesaria de la verdad de la proposición que q": "$Nc(p,q)$" $=_{\mathrm{df}}$ "$N(q\rightarrow p)\&M\sim p\&Mq$".

Se sugiere, después, una definición de "debe ser el caso que p": "Op" $=_{\mathrm{df}}$ "$Nc(p,I)$".

"I" es una constante proposicional cuyo contenido dejaremos abierto por el momento.

La exigencia, en la definición de Nc, de que $\sim p$ sea posible y de que q sea posible, que llamaremos *cláusula de contingencia*, es necesaria para evitar que "Ot" sea un teorema, y para garantizar que "$\sim O\sim t$" lo sea.

Se podría objetar que la cláusula de contingencia es una modificación *ad hoc* con la sola finalidad de acomodar la lógica deóntica dentro de la teoría de las condiciones. La objeción no es justa. La adición de la cláusula de contingencia es razonable por consideraciones que nada tienen que ver con los conceptos normativos y la lógica deóntica. Es cierto que permanece sin resolverse el problema de la condicionalidad de proposiciones necesarias e imposibles, pero eso debe ser explicado en una lógica modal de orden superior. Es muy fácil, además, transladar la cláusula de contingencia a la definición del operador O, lo que no podría ser objetado.

Obsérvese, de paso, que "$Op \rightarrow O(p \vee q)$" *no es* un teorema de la lógica deóntica que construimos. Se trata de la famosa paradoja de Ross. El hecho de que no sea un teorema no significa, sin embargo, que todos los problemas que ha traído esta paradoja y sus variantes, hayan de quedar resueltos. Porque, bajo una cláusula condicional que afirme que "$p \vee q$" no es necesaria, la fórmula puede derivarse.

El hecho de que la paradoja de Ross no sea derivable refleja una restricción en la validez del principio de distribución para el operador O. Del que una conjunción sea contingente, no se sigue que cada uno de los miembros de la conjunción sea contingente: puede uno ser necesario. La implicación "$O(p\&q) \rightarrow Op\&Oq$" sólo se sostiene bajo la cláusula condicional "$M{\sim}p\&M{\sim}q$". De la misma forma, la implicación "$Op\&Oq \rightarrow O(p\&q)$" tiene que estar sujeta a la cláusula condicional "$M(p\&q)$". Podemos resumir ambas cláusulas y obtener el siguiente teorema:
"$M{\sim}p\&M{\sim}q\&(p\&q) \rightarrow (O(p\&q) \leftrightarrow Op\&Oq)$".

Definamos "condición suficiente" como sigue:
"$Sc(p,q)$" $=_{df}$ "$N(p \rightarrow q)\&Mp\&M{\sim}q$".

Uno de los significados de "puede ser p", cuando quiere decir "p está permitido", se define como sigue:
"$Pp =_{df}$"$Sc(p,I)$".

El símbolo "P" será usado solamente en este sentido fuerte y no para el sentido débil de "puede", que corresponde a "${\sim}O{\sim}$".

De la misma forma en que se mostró la restricción en la distribución del operador O, se muestra el siguiente teorema:
"$Mp\&Mq\&M({\sim}p\&{\sim}q) \rightarrow (P(p \vee q) \leftrightarrow Pp\&Pq)$".

Se muestra que, de acuerdo con las definiciones, si algo que puede ser (Pp), de hecho es ,entonces todo lo que debe ser el caso, es. Sólo podemos gozar de aquello que está permitido, para decirlo con otras palabras, a condición de no violar en el mismo acto ninguna de nuestras obligaciones. Esta es la formulación en términos deónticos del principio general de las condiciones que dice: una condición suficiente de algo puede lograrse sólo si se cumplen también todas las condiciones necesarias del mismo hecho. Decir que algo está permitido en el sentido fuerte, sin mencionar los deberes relativos es, por tanto, hablar *elípticamente,* como sucede cuando decimos que algo es condición suficiente sin mencionar condiciones necesarias.

Es fácil demostrar el teorema "$Pp \rightarrow \sim O \sim p$". Es por esta razón que llamamos a un sentido de "puede" fuerte y a otro débil, porque el primero implica al segundo. Es en su sentido débil en el que este concepto aparece en los cálculos tradicionales de lógica deóntica.

Se examina después el famoso problema jurídico de las lagunas de la ley. ¿Es cierto que todo lo que no está prohibido está permitido? Kelsen piensa que sí, pero otros juristas creen que la clausura de un sistema jurídico depende de materias contingentes. La clave de la solución es la distinción de los dos sentidos de "permitido". Lo que no está prohibido, i.e., "$\sim O \sim p$", está obviamente permitido en el sentido débil. Pero "Pp" no se sigue lógicamente de "$\sim O \sim p$", de modo que no todo lo que no esté prohibido estará permitido en el sentido fuerte por necesidad lógica.

Si asumimos que la conjunción de todas las condiciones necesarias de determinada situación objetiva es condición suficiente de esa misma situación, nos encontramos con un caso de lo que tradicionalmente se ha denominado 'determinismo'. Ahora imaginemos un sistema legal en el que una vez que todas nuestras obligaciones estén cumplidas, cualquier cosa que hagamos sea legítima. Es claro que en este caso tendremos una "I" determinada, i.e., la conjunción de sus condiciones necesarias será condición suficiente de "I". Este sería, pues, un sistema normativo cerrado. La idea de la clausura de un orden legal es, por tanto, un caso especial del determinismo.

Uno de los conceptos que nunca habían podido definirse sin falsas implicaciones, era el de la obligación condicionada (commitment). En este trabajo se logra proporcionar una definición que salva todas esas dificultades. "Debe ser el caso que q, dado que p" queda, pues, definido como sigue:

"$Q(q/p)$" $=_{\mathrm{df}}$ "$Sc(p,Oq) \& \sim N(p \rightarrow q)$"

El hecho de que ocurra p *será,* pues, una condición suficiente para que deba ser p, pero no tendría sentido hablar de "obligación" si p fuera condición suficiente de q. En tal caso, al suceder p, sucedería q.

La incorporación de la lógica deóntica a la lógica de las condiciones proporciona también nuevas formas para lidiar con el problema de la repetición de operadores deónticos. Es claro que, si bien "$Op \rightarrow p$" no puede ser una verdad de la lógica, "$O(Op \rightarrow p)$" tiene que serlo, pues afirma que debe ser, que si algo deba ser, sea. En este trabajo se logra demostrar que esta verdad es un teorema.

En las definiciones que se han dado aquí de las nociones deónticas, no se ha especificado el contenido de la constante "I". Una sugerencia podría ser la de que no deba especificarse para definir "debe". De acuerdo con este punto de vista, decir que algo debe ser es decir que es condición necesaria de algo que se da por supuesto. Todo enunciado de la forma "debe ser p" sería, pues, típicamente elíptico. Esto nos hace claro la pertinencia de la pregunta "¿debo hacerlo? ¿para qué?" El crítico moral tendrá obviamente que hacer esa pregunta con respecto a "debes" aceptados.

Aun cuando aceptemos que la "I" de nuestra definición de "debe" puede ser cualquier cosa suceptible de tener condiciones necesarias, hay que dar especial atención a ciertos casos. La distinción entre *tipos* de casos se hace en parte en atención al contenido de "I", y en parte contemplando el tipo de necesidad involucrado en la relación condicional. Dos tipos importantes de casos que deben distinguirse son, primero, aquel en el que se toma "p" como un requisito lógico (conceptual) de "q" y, segundo, cuando se piensa que la necesidad es causal. Las estipulaciones relativas al primer tipo son o parecen *definiciones*. Las reglas o normas que se establecen de acuerdo con el segundo, pueden llamarse *normas técnicas* o *necesidades prácticas*.

El problema del contenido de "I" es de particular interés para la filosofía cuando nos encontramos en contextos morales o jurídicos. Se somete a consideración que, para un tipo muy importante del "debe" legal, "I" se caracteriza como *inmunidad al castigo* (a una reacción punitiva por parte de la maquinaria legal). Los actos de nuestros deberes jurídicos son *necesarios* para asegurar la inmunidad, pero que sean *suficientes* depende de que el orden jurídico sea o no cerrado. Pero la noción de inmunidad es un tanto problemática. No podemos decir que "inmunidad" significa que, a menos que "p" sea hecho, el castigo tendrá lugar. Puede suceder, por variadas razones, que no suceda el castigo al incumplimiento. Pero si decimos que su significado es que *no podemos* ser castigados legalmente si cumplimos "p", tendremos que explicar el sentido de "no podemos", y esta noción es modal y, por ello, sólo puede relacionarse con "debe". Esto, sin embargo, no implica circularidad, ni muestra que "inmunidad" no sirva para definir nociones de obligación legal. Sólo muestra que en la estructura del orden legal están involucrados conceptos no legales de obligación.

Los enunciados condicionales son enunciados genuinos, se les puede atribuir valor de verdad. Esto no quiere decir que hayan de ser "naturalistas", que deban verificarse necesariamente en la experiencia. Todo depende del tipo de condición de que se trate. Cuando se trata de normas técnicas, es claro que su confirmación será empírica, pero cuando tenemos necesidad lógica o conceptual, la verificación será la correspondiente.

Comúnmente se sostiene que las normas no tienen valor de verdad. Es necesario conciliar el carácter "teórico" de los enunciados condicionales con la "ateoricidad" de las normas. Resolver este problema es remover mucha de la confusión que prevalece en la filosofía de la moral y del derecho. Hay varias razones por las que parece que los enunciados normativos no son verdaderos enunciados. La primera debe buscarse en el carácter elíptico de la palabra "debe". Un desacuerdo sobre lo que debe ser, puede ser originado porque no se haya acordado el para qué. Aun cuando se reconozca el carácter elíptico de "debe", puede no tenerse clara la "*I*" en cuestión. Se nos ha enseñado a creer en un debe y no se nos explica el "*I*". La norma toma una apariencia "alógica". En otro uso característico, "debe" sirve para definir. Cuando digo que a una buena pluma no se le sale la tinta, estoy introduciendo una propiedad como requisito de que una pluma fuente sea buena. Otro uso de "debe" es el relativo a encausar la conducta. Este es el sentido *normativo* de la palabra "puede". Los *imperativos* se usan con igual propósito. Sería una confusión pretender que oraciones como "debe ser *p*" sean imperativos. Los imperativos no tienen valor de verdad. Pero sí puede decirse que "debe ser *p*" se usa muy comúnmente como imperativo, es decir, con el propósito de apremiar a la gente para que se conduzca de cierta forma.

Es absurdo disputar si "abre la ventana" y "debes abrir la ventana" significan lo mismo. Se podría convenir en usar la segunda sólo cuando se pueda contestar la pregunta ¿para qué? Pero nada impide que hagamos un uso descriptivo y uno performativo en el mismo caso.