DEDUCIBILITY IMPLIES *RELEVANCE?* A CAUTIOUS ANSWER (ON PROFESSOR ORAYEN'S CRITICISMS OF RELEVANT LOGIC)*

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I believe it to be an underlying principle of logical theory that when a correct reasoning is involved the conclusion cannot be just anything; what can be inferred depends upon what premises we have. We are faced, in a rather imprecise manner, with the requirement that the premises be relevant to the conclusion if we are to have a good inference. This is not, of course, a proof that deducibility implies relevance but an attempt to make explicit what I think is usually considered a necesary condition for a correct deduction.

But it is not altogether clear what kind of "relevance" should exist between premises and their conclusion. For instance, in propositional logic the kind of relevance Anderson and Belnap (A&B) are interested in is closely related to the notions of "using in a proof" and "variable-sharing". A is relevant to B iff B can be inferred from (not just under) A; that is, iff A *could be used* in a proof of B from A. For instance, A&B reject $A \rightarrow (B \rightarrow B)$ because they believe that A may be irrelevant to $(B \rightarrow B)$ in the sense that A is not used in arriving to $(B \rightarrow B)$. With respect to the requirement of variable-sharing, A&B believe that in order to infer B from A it is necessary that A and B have some common meaning content and since they also think that in propositional logic commonality of meaning is carried by commonality of propositional variables, they conclude that A and B should share at least one propo-

* I am indebted to Professor Orayen for his encouragement to write down these remarks. Without his attitude my own research would had not developed to its present state. His fair-mindedness makes him worry more about progress than about defending himself from objections.

sitional variable. Due to this, formulas of the form $(A \& A) \rightarrow B$ are rejected.

Orayen[1] implicitly accepts this notion of relevance. Perhaps he does so for the sake of argument but I think he has taken too much for granted. His analysis can be misleading since it does not consider other theoretical possibilities. What is discussed in Orayen's paper is:

1) Does deducibility *in the sense of classical logic* (CL) imply relevance *in the sense of A&B*? or rather,

2) Should the intuitive notion of deducibility (underlying our logical researches) which CL tried to capture, imply relevance *in the sense of A&B?*

To the first question both A&B and Orayen answer, "No", for as we know, variable-sharing is not a CL metatheorem. In fact, in standard propositional logic A can imply B even if A and B have no common variable; this may happen whenever A is a contradiction or B a tautology.

To the second question Orayen's answer is negative while that of A&B is affirmative. I think Orayen is right: A&B claim that normal intuitions support their requirement of "A&B-relevance" for "intuitive deducibility", but Orayen shows the existence of even more commonly felt intuitions (those supporting the rules used in Lewis' argument: Simplification, Addition and Disjunctive Syllogism) that lead us to accept some cases of deducibility without A&B-relevance. Against this strategy A&B had raised objections (centered on a criticism of the Disjunctive Syllogism) that Orayen seems able to cope with. I believe Orayen makes his point: there can be acceptable deducibility without implying A&B-relevance.

But in all this discussion we are no longer trying to find out whether CL is relevant, but whether it needs to be "A&B-relevant". A&B cannot be said to be wrong because of demanding that the premises be relevant to the conclusion, but rather because of believing that "relevance" has to mean "A&B-relevance". Orayen tries to prove the existence of some

[1] "*Deducibility* Implies *Relevance*? A Negative Answer", *Crítica*, vol. XV, No. 43, 44, April-August, 1983.

cases of deducibility without any relevance, but only succeeds in proving the existence of some cases of deducibility without *some* sort of relevance, namely A&B-relevance. He disregards that for every true deducibility relation, even that of CL, a relation closely connected with our intuitions about relevance must be involved. So it must be possible to trace some reasonable kind of relevance in CL although not necessarily the one A&B describe. I do not wish to diminish the importance of the notion of relevance depicted by A&B; I am just trying to argue that it is not the only possible one.

But, what kind of relevance can be found in CL?

To answer this I would like to state what I call "Ackermann's dictum": To say that from A we can deduce (in a strong sence) B is equal to saying that the content of B is a part of the content of A.[2] and this implies that A must be relevant to B.

There is at least one notion of propositional content that satisfies Ackermann's dictum with respect to classical deducibility. If we take the content of a proposition to be the set of state descriptions which falsify that proposition (following certain ideas of Popper and Wittgenstein) it is easy to show how the content of a tautology is part of the content of any proposition, which in turn is part of the content of any contradiction. The set of state descriptions which falsify a tautology is empty and therefore contained in the set of state descriptions which falsify any proposition whatsoever; and the set of state descriptions which falsify any proposition whatsoever is a part of the set of state descriptions which falsify a contradiction, since every state description does so. Therefore, e. g., A is relevant to $B \supset B$ and $A \;\&\; \bar{A}$ is relevant to B, if relevance is understood as the content relation described above. It is easy to see how the notion of relevance

[2] Cf. W. Ackermann, "Begründung Einer Strengen Implikation", *The Journal of Symbolic Logic*, vol. 21, number 2, June 1956, p. 113.

just sketched above holds for these well known cases of A&B-irrelevance.[3]

Do not be fooled by the oddity of saying that the content of $B \supset B$ is part of the content of any A whatever. Notice that the content of a complex proposition is not only determined by the propositions it contains but also by its syntactic structure. A&B believe that two propositional formulas share intensional content iff they share a variable and they do not realize that tautologies and contradictions are the extreme cases in which the content of a compound formula is not only affected but determined by the syntactic structure.

To summarize: it is misleading to say as Orayen does, that Lewis' argument is an excellent argument in favor of the existence of deduction without relevance (section 11.3). Lewis' argument shows that there is deducibility without A&B-relevance. It does not show that there is deducibility without any kind of relevance. This would shock our intuitions and I have hinted at least one sense in which CL satisfies these intuitions. Since I agree with Orayen that CL is correct in its inferences I take it to be unfair to say, without some nuances, that it validates some irrelevant arguments.

[3] After writing this comment I formally developed the notions of *propositional content* and *relevance* and proved a metatheorem showing that CL can satisfy Ackermann's dictum with respect to those notions. This is the content of a paper of mine prescnted at the IV Simposio Internacional de Filosofía (IIF-México), to be published in the proceedings of the forementioned Symposium.