DISCUSIONES

CONDITIONALS AND RELEVANCE. A RECONSIDERATION

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In my article 'Deducibility Implies Relevance? A Negative Answer'[1] I criticized the philosophical claims of relevant logicians but I also suggested that some systems of relevant logic may be useful for the study of conditional assertions of ordinary language. The critical aspects of the paper are developed in full, but the positive suggestions about conditionals are briefly stated. Because of this, a statement I made in two passages is either misleading or, even worse, false if taken literally. In this note I want to correct that statement and make two additional logical remarks about the subject.

In the passages in which I made the misleading remark, I was trying to make suggestions to improve a theory of ordinary conditionals advanced by Faris, that has both intuitive appeal and paradoxical consequences. Faris[2] maintains that the condition *E* that I am going to state below is a necessary and sufficient condition for the truth of a conditional of the form 'if *p* then *q*'. The condition *E* is stated as follows:

Condition *E*: There is a set *S* of true propositions such that *q* can be deduced from *S* with the addition of *p*.

In 'Deducibility (II)', pp. 20-1, I showed that Condition *E* has some intuitive support. But it has paradoxical consequences

1 Published in two parts, (I) and (II) in *Crítica*, vol. XV, no. 43, pp. 3-29, and *Crítica*, vol. XV, no. 44, pp. 3-25, respectively. I will refer to the article by the abbreviations 'Deducibility (I)' and 'Deducibility (II)', followed by the page number of the quoted passage.

2 J.A. Faris, *Truth-Functional Logic*, Routledge and Kegan Paul, London, 1962, pp. 117 and ff.

as well.[3] Interestingly enough, sometimes the counter-intuitive consequence is due to an inference that is accepted in classical logic but rejected in relevant logic. The proof that if p is false, it is true that if p then q, is such a case. For that proof, you take $S = \{-p\}$ ($-p$ is true if p is false). From S and p you can deduce q in classical logic (where *modus ponens* and the formula '$(p \,.\, -p) \supset q$' are valid). But that deduction is rejected in, for instance, relevant System E. So, you can avoid some counter-intuitive consequences of Condition E by imposing the restriction that in the deduction that is there mentioned only deductive inferences that are valid in System E[4] can be used. I will call this restriction 'R_1'. Other paradoxical consequences of Faris' Condition E are obtained when you deduce q from p and S but without using p throughout the deduction. For instance, you can prove in this way that if q is true, it is also true that if p then q. Here you take $S = \{q\}$. Obviously, you can infer q from S and p without using p. I gave another example (borrowed from Anderson and Belnap) in which something similar happens, in 'Deducibility (I)', p. 9. Here, when discussing the example, I made for the first time the incorrectly stated suggestion. The passage is as follows:

> *O* rejects the conditional 'If every signal has a maximum velocity, then there is a maximum velocity which no signal exceeds'. *E* makes him note that if the conditional is interpreted as 'material', *O* should acept it since he admits the truth of the theory of relativity and, therefore, the truth of the consequent. *O* makes clear that what he meant was that 'there was no way of arguing correctly from the antecedent to the consequent'. *E* suggests then that, when trying to make such a deduction, we would be able to use, added to the antecedent, other relevant premises. *O* assents and *E* gives the death blow: using also Einstein's theory we *can*

[3] These consequences are the same as those that arise from acceptance of the idea that the truth-table of '$\supset$' is valid for the conditional of the ordinary language.

[4] It is only a coincidence that Faris' Condition and Anderson and Belnap's System have the same name: '*E*'. To avoid confusion about what I am referring to I will always use the word 'Condition' or 'System' before '*E*'.

deduce the consequent from the antecedent. But that is possible only if we are using the "official" theory of deduction. If relevance restrictions are imposed, the deduction will not be legitimate because, in the analized inference, the consequent is deduced from the antecedent and other premises (extracted from the theory of relativity), but the antecedent itself is superfluous, since really only the premises added are used in the process. In that case, one of the premises used is irrelevant to the conclusion and that invalidates the reasoning from the point of view of relevant logic.

The final sentence of the quoted passage seems to state that if one of the two premises from which a conclusion is obtained is not used in the deduction, the reasoning from the two premises to that conclusion is wrong from the viewpoint of relevant logic. If that were the case, R_1 would be enough to impede the proof of the conditional in the last example by using Condition *E* (because the consequent is deduced from the antecedent and the theory of relativity –taken as *S*– but the antecedent is not used in the deduction; something similar happens in the proof that if *q* is true, so also is that if *p* then *q*). But, interpreted in such a way, the last sentence of the passage is obviously wrong: if you can deduce *r* from *p* and *q* by using deductive rules allowed in System *E*, the inference *p*, *q* / *r* is valid in that system even if you did not use *p* in the deduction. For, let us suppose that *q* alone implies *r*. In that case, it is true that

(1) $q \rightarrow r$ ('$\rightarrow$' means here the same as 'logically implies that')

But, by the axiom E5 of System *E*[5] it is valid that

(2) $(p \,\&\, q) \rightarrow q$

[5] For the rules I am referring to in this passage, see Anderson and Belnap, *Entailment*, Princeton University Press, vol. I, 1975, p. 232.

and because of rule &I of the same system, the following inference is valid:

(3) p , q / p & q

By the transitivity of deducibility –accepted by Anderson and Belnap–, (3) and (2) show that p , q / r is valid from the point of view of System *E*, if you can deduce *r* from *q* by using rules valid for relevant logic.

What I had in mind when I wrote the quoted passage of 'Deducibility (I)' was that in order to avoid proofs of conditionals like that in the last example, it may be a good idea to put a restriction on the statement of Faris' Condition *E*, similar to restrictions that in the Fitch-style formulation of *E* impede the proof of formulas like '$B \rightarrow .A \rightarrow A$'. When Anderson and Belnap discuss the proof of that formula in other systems, they say that "... this fact suggests a solution to the problem: we should divise a technique for keeping track of the steps used, and then allow application of the introduction rule only when *A* is relevant to *B* in the sense that *A* is *used* in arriving at *B*."[6] After this passage, Anderson and Belnap devise a technique of the required kind. The difference from my misleading statement is that, to block the proof of '$B \rightarrow .A \rightarrow A$', they don't consider invalid a deduction of *A* from *B* and *A*, when *B* is not used in the deduction; that deduction may be valid, but what is invalid is the inference *from that deduction* that *B* can be introduced as an antecedent, and it is here where the restriction works. In a similar way, what I should have proposed is that, to avoid proofs of conditionals like that in the quoted passage of 'Deducibility (I)', the following restriction R_2 in Condition *E* must be introduced:

R_2 It must be possible to deduce *q* from *p* and *S* in a deduction in which *p* is actually used in arriving at *q*.

I will finish with two remarks. The first one is that R_2 is not enough to block the proof of counter-intuitive conditionals

6 Anderson and Belnap, *op. cit.*, p. 18.

like those we have analysed in the two last examples. It is necessary to reinforce R_2 with R_1. I will show this by proving that the truth of a conditional with a true consequent follows from Condition *E* even if it obeys R_2. This is made possible by using the antecedent of the conditional in a trivial way within the deduction. Both the next two deductions can be used with that purpose:

I:

(1) p	Hyp.
(2) $-p \vee q$	Hyp.
(3) $--p$	(from 1, by Double Negation)
(4) q	(from 2 and 3, by Disjunctive Syllogism)

II:

(1) p	Hyp.
(2) $p \supset q$	Hyp.
(3) q	(from 1 and 2, by *Modus Ponens*)

(Note that in both the two deductions, (2) is true if q is true.)

Either I or II proves that if q is true, so also is that if p then q, by using Condition *E* with R_2. In I, $S = \{-p \vee q\}$; in II, $S = \{p \supset q\}$. In both cases, we deduce q from p and S and p *is used* in the deductions. But, if we use Condition *E* with R_1 *and* R_2 instead, I and II cannot be used for these proofs, due to the fact that Disjunctive Syllogism and *Modus Ponens* are not valid in System *E*.

The second remark is that by stating R_2, I am not giving a precise restriction but only pointing to the direction in which a solution could be achieved. It would be necessary to work out formally the restriction to see whether it is efficient. There are some clues suggesting that Anderson and Belnap were planning to develop theories of conditionals of this sort in the second volume of *Entailment*. We shall see what can be done in this respect.