EXTENSIONALITY AND PROPOSITIONAL IDENTITY

A. N. PRIOR
Oxford University

1. *The fallacy of extensionalism.* Certain logicians have attempted to simplify their systems by saying that if the sentences S1 and S2 have the same truth-value then any compound sentences which differ only in one having S1 where the other has S2 must have the same truth-value also. If this "law of extensionality" were true, and "X thinks that grass is pink" and "X thinks that grass is purple" were genuine compounds with "Grass is pink" and "Grass is purple" as components, then these compounds would have to have the same truth-value since the corresponding components do (both being false). It is, however, plain that a man may think that grass is pink without thinking that grass is purple. The moral drawn from this is that "X thinks that grass is pink" is not a genuine compound with "Grass is pink" as a component, or as it is technically put, not a genuine function with "Grass is pink" as argument. But I cannot see the slightest reason, other than stubbornness, for not drawing the moral that the law of extensionality is false. There is, it is true, a large and interesting area of logical theory within which it holds, just as there is a large and interesting area of physical theory in which we can retain the laws of classical mechanics; but I cannot see the least reason for claiming any more for it than that. I have been assured by some of its defenders that they can see immediately and intuitively that it is true; I can only say that such intuitions as I personally have about the matter are all to the contrary.

It is sometimes said, particularly by logicians of the schools of Lesniewski and Lukasiewicz, that if the law of extensionality is abandoned we must admit that some propositions are neither true nor false, i.e. that for some p, it neither is the case that p nor is not the case that p. The argument for this is so riddled with confusions that it is painful to have to examine it; however, its proponents include logicians of such distinction that we had better go through with it. If all propositions are either true or false, it is argued, there can only be four propositional functions of one propositional argument — one, call it Vp (for "Verum p"), which is true as a whole whether its argument p is true or false; a second, call it Fp (for "Falsum p"), which is false as a whole whether p is true or false; one, call it Np (for "Not p"), which is true if p is false and false if p is true; and one, call it Sp (sometimes read "Assertium p", or "It is the case that p"), which is true if p is true and false if p is false. And it is true of each of these functions that if p and q have the same truth-value then that function of p will have the same truth-value as that function of q. This can easily be verified in each case. Vp and Vq, and Fp and Fq, have the same truth-value (truth in the first case and falsehood in the second) whatever the truth-values of p and q may be. If p and q are both true, Sp and Sq will be both true, and Np and Nq both false; and if p and q are both false, Sp and Sq are both false, and Np and Nq are both true. So if any function does not obey the law of extensionality, it cannot be one of these four, and if there are other functions beside these, there must be more possible truth-values to generate them.

The very first step of this argument assumes what it claims to prove, namely that the only feature of p on which the truth-value of any function of it can depend is p's own truth-value. For the list of possible functions simply does not include any which are, say, true with some true arguments and false with other true arguments. If "X thinks that p", for example, were a function of p, it would have precisely this character. Why on earth should not the truth-value of a function of p depend on some other feature of p than p's truth-value? To say that this is impossible, is like saying that for any genuine function fx of a number x, whether fx is greater than O *must* depend on whether x is greater than O — an assumption which is plainly false, for example, if the function is $x - 1$; since in some cases when x is greater than O, e.g. when $x = 2$, $x - 1$ is also greater than O, whereas in other such cases, e.g. when $x = 1$, $x - 1$ is *not* greater than O. Whether *this* function of x is greater than O clearly depends not on whether x itself is greater than O, but on whether it is greater than 1. Similarly, whether it *is or is not the case* that X believes that p does not depend on whether *it is or is not the case* that p, but on whether *it is or is not believed by* X that p. Why on earth not?

2. *Frege on functions and values.* I suspect that the real villain of this piece is Gottlob Frege, and indeed that the rot set in with his invention of the term "truth-value", in the mathematical setting which gave it its original meaning. Being greater than O is not, of course, strictly speaking the "value" of a numerical function for a given argument; its value for that argument is not a property of a number (such as being greater than O), but a number. For example, the value of the function $x - 1$ for the argument 2 is 1, and for the argument 1 it is 0, and it has not a whole collection of values with a given argument in the way that it has a whole collection of properties with a given argument (when $x = 2$, for example, $x - 1$ is greater than 0, less than 3, its own square, and so on). And Frege held that sentences designated or denoted objects called Truth and Falsehood in the same way that numerals, and formulae containing numerals, designate or denote numbers. What number is denoted by a given numerical function-expression does depend on what number is denoted by its argument-expression (or expressions), and on nothing else. Hence, if the parallel holds, which out of Truth and Falsehood is denoted by a given

sentential function must depend on which of them is denoted by its argument-sentence, and on nothing else. That it is not the case that grass is pink, we might say, not merely "is true", in the way that $2 - 1$ is greater than 0 (and also is other things, e.g. its own square), but *is Truth,* in the way that $2 - 1$ *is the number 1* (and is nothing but the number 1). And that-it-is-not-the-case-that-grass-is-pink *is Truth* just because that-grass-is-pink *is Falsehood,* just as $(1 + 1) - 1$ is the number 1 just because $1 + 1$ is the number 2. Equally, that-it-is-not-the-case-that-grass-is-purple *is Truth* just because that-grass-is-purple *is Falsehood,* just as $(3 - 1) - 1$ is the number 1 just because $3 - 1$ is the number 2. And the "Truth" which that-it-is-not-the-case-that-grass-is-pink "is", is the very same "Truth" which that-it-is-not-the-case-that-grass-is-purple "is", just as the number 1 which $(1 + 1) - 1$ "is" the very same number 1 which $(3 - 1) - 1$ "is". There are not several Truths which different sentences might denote, any more than there are several number-1's denoted by such different expressions as "$(1 + 1) - 1$" "$(3 - 1) - 1$".

If Frege's parallel holds, all this follows. But of course it doesn't hold, and truth and falsehood are much more like properties (to be set alongside other properties of what sentences denote, than themselves what sentences denote. They are not quite that, certainly; for sentences do not in fact denote anything, and propositions are things-with-properties only in a Pickwickian sense. But we know enough by now, all the same, to make this sense quite precise, and to use the things-with-properties locution harmlessly. For the matter of that, numbers are not things-with-properties (denoted by numerals) either, but here too we know enough now to use these locutions harmlessly — we know what we mean, e.g. if we say that 1 is greater than 0, viz. that for any φ and ψ, if exactly one thing φs and nothing ψs, then more things φ than ψ. And there are innumerable things that we can "say about propositions" in the sense in which we can say about them that they are true or that they are false, just as there are innumerable things that we can "say about numbers" in the sense in which we can say about them that they are or that they are not greater than 0.

Frege himself knew that there was more to be said about the functioning of sentences than that they designate Truth or Falsehood, but said it so clumsily that it has been ignored or denied by those who have most eagerly taken up the other. For he spoke of a "sense" as well as a "denotation" of sentences, and acknowledged that there were genuine functions of a sentence's sense as well as of its denotation; one such function of the sense of "Grass is pink" being that expressed by the sentence "X thinks that grass is pink". But this is to make a dichotomy where there is none. What X thinks when he thinks that grass is pink, is precisely that whose not being the case makes this thought a false one — it is certainly not the case that there is one thing, the denotation of the sentence "Grass is pink", which is not the case ("is Falsehood"), and quite another thing, the "sense" of this sentence, which is believed by X. To have a false belief is to believe precisely what is not the case, not to believe something else which is merely connected in some obscure way with what is not the case. Truth-functions and belief-functions, in short, are functions of the same arguments; we must resist above all things the madness which insulates what we think from any possibility of directly clashing with what is so.

Frege's theory, to be fair, does not do quite that, and there would seem to be a possibility of constructing something like the ordinary theory (i.e. the one developed here) within this. For it would seem that the *Gedanken* or "propositions" which constitute the sense of sentences do have, beside such properties as that of being thought by X, the property of being the sense of what denotes Truth or Falsehood (or, as Frege allowed, in certain cases, neither) as the case may be; or as Church puts it, the property of "being a concept of" Truth or Falsehood (or, if we were to follow

Frege here, neither). We thus have among the functions of Frege's *Gedanke* a set of functions related to one another in exactly the same ways as his functions of the True and the False are related to one another, and we can shear off these last as a superfluity. But the stone which we thus reject, the extensionalists have made the head of the corner.

I should add, though, that however misleading it may be to speak of sentences as denoting or designating truth-values in the sense in which proper names denote or designate individuals, it is not at all misleading, and is highly illuminating, to say that sentences have truth-values for their "extension", in the way in which predicates have classes for theirs. It will be best to draw out this parallel after something has been said about propositional identity, but it may be noted now that part of the parallel consists in the fact that neither truth-values *nor* classes are genuine objects, both being "logical constructions", and very similar logical constructions.

3. *Equivalence and propositional identity*. Pure extensionalism, i.e. extensionalism unmitigated by Frege's distinction between sense and denotation, in effect equates identity of what sentences mean with identity of their truth-value, i.e. with what is sometimes called their "material equivalence". For what extensionalists say about material equivalence, i.e. that all functions of materially equivalent sentences are materially equivalent, is really true of identity of meaning, or if we like to put it that way, of identity of the propositions which our sentences express. If the proposition that p really is the very same proposition as the proposition that q, then certainly any function of the proposition that p is the very same proposition as that function of the proposition that q. For example, if the proposition that *all bachelors are unmarried* really is the very same proposition as the proposition that *all unmarried men are unmarried*, then the proposition that *Jones wonders whether all bachelors are unmarried* is

the very same proposition as the proposition that *Jones wonders whether all unmarried men are unmarried.*

This last contention may seem questionable — perhaps, even, almost as questionable as the law of extensionality itself — but before tackling that one, let us consider a more far-reaching objection to this way of talking, namely that once we start talking about propositional identity we are committed to abandoning the view that propositions are logical constructions, and to treating them as genuine objects. Identification, like quantification, may be said to involve an "ontological committment" to the staright-forward objecthood of what is identified. But it is no more clear that this is so in the case of identity than it is in the case of quantification.

Suppose we write *Ipq* for "The proposition that *p* is the very same proposition as the proposition that *q*". The apparent names "The proposition that *p*" and "The proposition that *q*" just do not occur in the complex *Ipq*, which only has the functor or operator I and the sentences *p* and *q*, and in the verbal version these same apparent names can be considered as having no meaning or function outside of the entire complex.

(1) "The proposition that — is the very same proposition as the proposition that —"

(which is what we abbreviate to "*I* — — "), where the gaps are not for names but for sentences.

The form is exactly on a par with

(2) "The proposition that — implies the proposition that —",

which is no more than a fluffed-up way of writing

(3) "If — then —",

where these apparent names do not occur. The only difference is that we have no colloquial form analogous to (3) by which we can translate (1), but this is no more than an accident of language.

As to the *laws* of propositional identity, the fundamental ones are just *Ipp*, "Every proposition is identical with itself", and the one already mentioned, that all functions of identical propositions are identical. If, using the symbolism of Lukasiewicz, we write $C\alpha\beta$ for "If α then β", and use d as a variable standing for expressions which form a sentence out of a sentence, we may write this second law as *CIpqIdpdq*.

We are now in a position to examine objections to this law, and we begin with one hinted at earlier. It may well seem plausible to say that the proposition that all bachelors are unmarried is the very same proposition as the proposition that all unmarried men are unmarried, and yet that the proposition that Jones wonders whether all bachelors are unmarried is not the very same proposition as the proposition that Jones wonders whether all unmarried men are unmarried. We have the feeling that the second wondering would indicate much greater stupidity in Jones than the first wondering would. And precisely because there has been considerable argument among philosophers on this point, we are inclined to assent to the proposition that

(1) Many philosophers wonder whether the proposition that *Jones wonders whether all bachelors are unmarried* is the very same proposition as the proposition that *Jones wonders whether all unmarried men are unmarried,*

even though we are not at all inclined to assent to the proposition that

(2) Many philosophers wonder whether the proposition that *Jones wonders whether all unmarried men are unmarried* is the very same proposition as the proposition that *Jones wonders whether all unmarried men are unmarried.*

But if the law *CIpqIdpdq* is true, and if the proposition that all bachelors are unmarried is the very same proposition as the proposition that all unmarried men are unmarried, then the proposition that (1), which seems true, must be the very same proposition as the proposition that (2), which seems false. For (1) and (2) are just *dp* and *dq*, with "All bachelors are unmarried" for p, "All unmarried men are unmarried" for q, and for d the functor

> Many philosophers wonder whether the proposition that Jones wonders whether —, is the very same proposition as the proposition that Jones wonders whether all unmarried men are unmarried.

There is here, I suspect, a confusion between wondering whether all bachelors are married, and wondering whether what is expressed by the sentence "All bachelors are married" is true. And by "wondering whether what is expressed by the sentence 'All bachelors are unmarried' is true" I do *not* mean wondering, with respect to what is expressed by the sentence "All bachelors are unmarried", whether it is true; for this is indeed the very same thing as wondering whether all bachelors are unmarried. What I mean by it is not this but wondering, with respect to the sentence "All bachelors are unmarried", whether what it expresses is true. And a man might well wonder about this *without* wondering, with respect to the sentence "All unmarried men are unmarried", whether what *it* expresses is true. For a man might very well not know that what the sentence "All bachelors are unmarried" means is simply that all unmarried men are unmarried; and part of his reason for wondering whether what it expresses is true might be that he is wondering exactly what it is that it expresses. And in the case of our (1) and (2), this confusion may affect either or both of the outer and inner wonderings. That is, our wondering philosophers may really be wondering whether the proposition that Jones wonders whether what is expressed by the sentence "All bachelors are unmarried" is true, is the very same proposition as the proposition that Jones wonders whether what is expressed by the sentence "All unmarried men are unmarried" is true. Or again, they may be wondering whether the proposition expressed by the sentence "Jones wonders whether all bachelors are unmarried" is the very same proposition as the proposition expressed by the sentence "Jones wonders whether all unmarried men are unmarried", i.e. they may be wondering, with respect to these two sentences, whether they express the same proposition. There are other possibilities here also: but to go into them further would be tedious, and enough has been said to make our final answer clear. If there are *no* confusions of this sort, and if the person who is actually propounding the sentences (1) and (2) is himself using the sentence "All bachelors are unmarried" simply to mean that all unmarried men are unmarried, and if this person is using the form "x wonders whether p" to describe, not a wondering about a sentence but a wondering as to what is the case — a wondering (in Jones's case) about the unmarriedness of bachelors, *i.e. of unmarried men* — then this person is *ipso facto* using (1) and (2) to mean the very same thing.

4. *The "quotation-marks" objection.* There is, however, a much more substantial objection to the law *CIpqIdpdq*, if the variable *d* is taken to stand for *any* expression that forms a sentence from a sentence. For one way of forming a sentence from the sentence "All bachelors are unmarried" is to put quotation-marks around it and prefix "Jones uttered the sentence" to the result; i.e. one expression which forms sentences from sentences is "Jones uttered the sentence '. . . .' ", where the inner quotation-marks are part of the sentence-forming expression I mean. The quotation-marks themselves, of course, do not form a sentence from the sentence inside them; rather, they form — if anything — the *name* of the sentence inside them; but the whole expression consisting of "Jones uttered the sentence" plus the following quotation-marks, does seem to form a sentence from the quoted sentence. And it is quite clear that the proposition that Jones uttered the sentence "All bachelors are unmarried" is *not* the very same proposition as the proposition that Jones uttered the sentence "All unmarried men are unmarried". It is even clearer that the proposition that the second word in

the sentence "All bachelors are unmarried" is the word "bachelors", is not the same proposition as the proposition that the second word in the sentence "All unmarried men are unmarried" is the word "bachelors"; though the sentence by which we have expressed the former proposition, and the sentence by which we have expressed the latter, are constructed by wrapping one and the same expression around the sentence "All bachelors are married" in the one case and around the sentence "All unmarried men are unmarried" in the other. (I owe this objection to Professor C. Lejewski.)

These counter-examples are trivial and, one feels, sophistical, and yet they are difficult to exclude by any very clear rule or principle. It won't do to say, for example, that *CIpqIdpdq* holds so long as the expression for which *d* stands isn't one that finishes up (i.e. "finishes up" at the end or in the middle —wherever the argument-sentence is inserted) with quotation-marks. For (a) there are expressions which don't finish up thus for which the law just as obviously doesn't hold, and (b) there are expressions which do finish up thus for which the law fairly obviously does hold. For example —under (a)— "That all unmarried men are unmarried may be thus expressed to bring out its tautological character" surely does not express the same proposition an "That all bachelors are unmarried may be thus expressed to bring out its tautological character" (cf. Quine's example in a different connexion: "Giorgione was so called because of his size"). Perhaps one could say there are "implicit quotes" here —"thus expressed" means in the one case "expressed by 'All unmarried men are unmarried'", and in the other "expressed by 'All bachelors are unmarried'". More importantly, there is, in "thus expressed", a quite *ex*plicit reference to the form of words employed; the relevance of this will be indicated later.

As to expressions which finish up with quotation-marks for which the law seems nevertheless to hold: (1) "Jones said something with the sense of 'All bachelors are unmarried'" might very well express the same proposition as (2) "Jones said something with the sense of 'All unmarried men are unmarried'", and quite certainly expresses a proposition with the same truth-value, if the quoted sentences do indeed have the same sense or express the same proposition. It might be argued that (1) and (2) cannot express the same proposition because their translations into French are different, and are so regardless of whether or not French resembles English in being equally able to refer to unmarried men by one word ("bachelors") or by two ("unmarried men"). For, the argument runs, the French translation of (1) is "Jones a dit quelque chose qui a le sens de "All bachelors are unmarried'", whereas that of (2) is "Jones a dit quelque chose qui a le sens de 'All unmarried men are unmarried'". But this logicians' convention of not translating quoted expressions is by no means always followed in ordinary translation— it is proper enough when it is *quite* clear that we are only thinking of the quoted words *qua* sounds or marks (e.g. "'House' has five letters" must certainly be translated "'House' a cinq lettres" and not "'Maison' a cinq lettres"); but in e.g. translating a novel with a great deal of quoted conversation in it, it would be a pour translation which left all this in the original language.

In any case, there is something wrong in principle with the proviso we have suggested, depending as it does upon the presence or absence of a particular linguistic device, which might have had quite other uses, and which has a variety of uses even in ordinary English, and in other natural languages. The suggestion in fact involves a confusion between two very different ways of looking at such formulae as *CIpqCdpdq*. We may think of it, in the first place, as part of a rigorously formalised calculus; such a calculus will of course contain rules as to what we may substitute for the variable *d* and still preserve theoremhood; but these permitted substitutions will not be expressions of ordinary English but expressions belonging to the calculus to which *CIpqCdpdq* itself belongs. These expressions may or may not include devices which function as quotation-marks, or phrases ending with quotations-marks, sometimes do in English; if they do not, our difficulty doesn't arise; but of course if (in developing our calculus in detail) we deliberately see to it that they do not, we should be able to give reasons for such a course, and not just the reason that we would avoid trouble this way. The reasons must, in fact, be connected with the *other* way of looking at such a formula as *CIpqCdpdq*, to which we way now turn.

Even as part of a rigorously formalised calculus, the formula *CIpqCdpdq*, like any other formula expressing a law of logic, may be used to *say* something. Or perhaps more accurately, its "closure", i.e. the result of binding all the variables in it by initial universal quantifiecs, may be used to say something: i.e. *this*:

(1) $\pi p \pi q \pi d CIpqCdpdq$,

or in semi-English,

(2) For all *p*, for all *q*, for all *d*, if the proposition that *p* is the very same proposition as the proposition that *q*, then if *dp* then *dq*, may be used to say something. And what these are used to say in logic, is in general *not* something *about* sentences or expressions at all. (2) may be partially instantiated, for instance, by

(3) For all *p*, for all *q*, if the proposition that *p* is the very same proposition as the proposition that *q*, then if someone brings it about that *p* then someone brings it about that *q*.

And this in turn may be instantiated by

(4) If the proposition that all bachelors are fined £20 is the very same proposition as the proposition that all unmarried men are fined £20, then if someone brings it about that all bachellors are fined £20, then someone brings it about that all unmarried men are fined £20.

In (1), (2) and (3) it would in fact be natural to put an "*ipso facto*" after the last "then", e.g. to alter (4) to

(5) If the proposition that all bachelors are fined £20 is the very same proposition as the proposition that all unmarried men are fined £20, then if someone brings it about that all bachelors are fined £20, then *ipso facto* someone brings it about that all unmarried men are fined £20.

Such an addition perhaps amounts to strengthening the second *C* in (1) to an *I*, i.e. the proposition that (5) is perhaps the very same proposition as the proposition that

(6) If the proposition that all bachelors are fined £20 is the very same proposition as the proposition that all unmarried men are fined £20, then the proposition that someone brings it about that all bachelors are fined £20 is the very same proposition as the proposition that someone brings it about that all unmarried men are fined £20.

However this may be, it is important to notice that when we pass from (2) and (3) to (4) the subordinate sentences which finally replace *dp* and *dq* (in our example, "someone brings it about that all bachelors are fined £20" and "someone brings it about that all unmarried men are fined £20") are not *about* the sentences which finally replace the p and q (in our example, the sentences "all bachelors are fined £20" and "all unmarried men are fined £20"), but they are about whatever these latter sentences are about— in our example, the larger sentences, like the smaller ones, are about bachelors, i.e. unmarried men, and about what happens to such beings. (Indeed (4) in its entirety is about bachelors, in the sense in which "All bachelors are bachelors" is about them.) And what is intended by (1) and (2) is something tha has *only* this sort of instantiation; we do not count as instantiations (for what we mean by (1) and (2) is something that does not *have* as instantiaitions) cases in which the larger sentences are *not* about what the smaller sentences are about, but are rather about those sentences themselves. Pseudo-instantiations of the latter sort are excluded *whether or not* we are using a language in which we sometimes talk about sentences by first actually writing these sentences down (rather than spelling them, Gödelnumbering them, etc.) and then surrounding them with quotation-marks and their predicative appendages.

This position does not depend on any special view of *how* quotation-marks contribute (when they *are* being used this way) to talking about sentences themselves instead of about what the sentences are about. In many circles the stock view is that the quotation-marks plus the enclosed sentence constitute a *name* of the sentences enclosed; but this may be and has been disputed. Some would say that the quotations do not go with the enclosed sentence but rather with what is on the *other* side of them to form a peculiar sort of predicate, like "'—' contains nineteen letters", or "John uttered the sentence'—'". Some, again, would say that the quotation-marks are *demonstratives* which *point* to their interior, so that "'The cat sat on the mat' has nineteen letters" is rather like "The cat sat on the mat. This has nineteen letters". I incline to this view myself; and certainly if it is the correct view it is easy to classify the illusion involved in treating "'The cat sat on the mat' has nineteen letters" or "'The cat sat on the mat' was uttered by John" as compound sentences with "The cat sat on the mat" as a component. This is simply the illusion of seeing two sentences as one, because they happen to stand in an interesting relation to one another.

One final word about this difficulty. Nothing could be more misleading and erroneous than to treat sentences containing (if that is the right word) other sentences in quotation-marks as a paradigm case to which the things that we are really interested in (thinking that, fearing that, bringing it about that) should be assimilated. It is a completely off-beat case which, having mentioned, we are entirely justified in forgetting.

5. *Truth-values and classes.* We may now turn to the parallel between truth-values and classes which was hinted at earlier. To see this parallel accurately, something must first be said about what talk about classes really amounts to. The most elementary form of statement that is ostensibly about a class is a statement to the effect that something is a member of it, i.e. a statement of the form "x is a φ-er" or "x is a member of the class of φ-ers". Such a statement is only ostensibly about a class, since it amounts to no more and no less than the simple statement "x φs" (from which the appearance of being about a class as well as about x has vanished). More complicated statements which are ostensibly about classes can all be defined in terms of this elementary form. For example, "The class of φ-ers is included in the lascs of ψ-ers" simply means "Whatever is a member of the class of φ-ers is a member of the class of ψ-ers", i.e. "Whatever φ's ψ's".

But this is not the whole story. The complications come in when we wish to *count* classes. The notion of counting is bound up with that of identity. To say that exactly one individual φs is to say that for some x, x φs, and for any x and y, if x φs and y φs then x is the same individual as y. To say that exactly two individuals φ is to say that for some x and y, x φs, y φs, and x is not the same individual as y, and for any x, and z, if x φs, y φs and z φs, then either x is the same individual as y, or y as z, or z as x. The counting of propositions and properties involves higher-level quantifications and identifications of the same sort. "Prior and Quine are agreed on exactly one thing", for example, would expand to

> For some p, both Prior and Quine believe that p, and for any p and q, if both Prior and Quine believe that p, and both Prior and Quine believe that q, then the proposition that p is the same proposition as the proposition that q,

where the last clause is understood as in the preceding section. And analogously to our understanding of that clause, we may understand the form

The property of φ-ing is the same property as the property of ψ-ing,

or more briefly

To φ is the same thing as to ψ,

as constructed out of the verbs φ and ψ by a two-place functor with one-place verb arguments, from the same syntactical box as the functor

Whatever ( )s ( )s,

though it is not the same functor. The component "The property of ( )ing" is not to be understood as forming a noun from the verb that goes into the gap, but as an inseparable part of the whole functor, so that the use of this form is not to be understood as committing one to the existence of properties any more than one is committed to this by the use of the form "Whatever φ's ψ's". This being understood, "There is exactly one property which applies to nothing" may be expanded to

For some φ, nothing φ's, and for any φ and ψ,
if nothing φ's and nothing ψ's, then to φ is
the same thing as to ψ.

This is, of course, untrue; to be a mermaid, for example, is not the same thing as to be a centaur. On the other hand, the *class* of mermaids *is* the same class as the class of centaurs; there *is* exactly one "ampby class" or "null class".

This situation is sometimes misdescribed by saying that classes and properties are different entities. Misdescribed, because neither classes nor properties are entities at all. The real difference is that between the meaning of "Exactly *n* proprties Φ" and "Exactly *n* classes Φ". The former is defined in terms of quantification over verbs and verb-identity, while in the latter, verb-identity is replaced by common application, or what Russell calls "formal equivalence". To say that the properties of φ-ing and of ψ-ing are "formally equivalent" is simply to say that whatever φ's ψ's and whatever ψ's φ's. And to say that the class of φ-ers is identical with the class of ψ-ers is simply to say that the properties of φ-ing and of ψ-ing are formally equivalent. Hence to say that there is exactly one null class is to say that

For some φ, nothing φ's, and for any φ and ψ, if nothing φ's and nothing ψ's, then whatever φ's ψ's and whatever ψ's φ's,

which is true. It is convenient to talk *as if* there were entities, classes, which are identical when their defining predicates apply to the same objects, but in fact to say that these entities are identical just *is* to say that these predicates apply to the same objects, and this in turn just *is* to say, of the given φ and ψ, that whatever φ's ψ's, and whatever ψ's φ's.

Two-place predicates may be associated with "relations in extension" in an exactly similar way. For example, to be both-father-and-mother-of is not the same as to be both-taller-and-shorter-than, but the corresponding "relations in extension" *are* the same; for this is just to say that for any x and y, if x is both father and mother of y then x is both taller and shorter than y, and *vice versa* (the two implications being of course vacuously true, since no objects are related in either of these ways). Symbolically, identity of the class of φ-ers with the class of ψ-ers is expressed by

(1) $\pi x E\varphi x\psi x$,

and identity of the relations-in-extension corresponding to φ-ing and ψ-ing by

(2) $\pi x\pi y E\varphi xy\psi xy$.

Similarly again, identity of the extensions of the three-place predicates φ and ψ is expressed by

(3) $\pi x\pi y\pi z E\varphi xyz\psi xyz$.

We can clearly go upwards from this as far as we like, but we can also go downwards from (1) one further. It φ and ψ are "*no*-place predicates", i.e. complete propositions, we can say that their extensions are identical when we have

(0) $E\varphi\psi$,

or in words "If φ then ψ and if ψ then φ". This is the case when and only when either it both is the case that φ and is the case that ψ, or is not the case that φ and is not the case that ψ.

So we concoct the terms "truth-value" for what we will describe as identical when condition (O) is met, just as we concoct the term "class" for what we will describe as identical when condition (1) is met, and "relation in extension" for what we will describe as identical when condition (2) is met. But we ought never to forget that these assumptions of identity mean no more and no less than the statement of the "conditions" (0), (1) and (2).

The identity-conditions for "numbers", it may be added, are analogous. "For no x does x φ" is not the same function of φ as "For all x, if x φs then x is not identical with x", but they apply to exactly the same φ's, i.e. we have

(4) For any φ, (for no x, x φ's) if and only if (for all x, if x φ's then x is not identical with x).

Hence, if we abridge "For no x, x φ's" to "$Ox\varphi x$" and "For all x, if x φ's then x is not identical with x" to "$0^1x\varphi x$", we may say that 0 and 0^1 are "the same number".

These odd uses of "same", and inventions of entities for them to apply to, are of the utmost symbolic convenience, but failure to understand just what is going on when we avail ourselves of these devices has been philosophically disastrous. For example, it is fashionable to talk as if we have no right to speak of the identity of this or that unless we can formulate the "conditions of identity" for the entities or quasi-entities in question. This demand is entirely in order when we are, as above, indulging in extensional abstraction, i.e. when what is involved is not *really* identity at all, and when it is therefore necessary to exaplain what surrogate for identity we are employing; but in most other cases the demand is a senseless one (identity, like quantification, just is what it is and not another thing). The error to which the present chapter is principally devoted —the treatment of truth-values as the *denotator* of sentences, and the confusion of propositional identity with material equivalence, is a special case of this more general muddle.

RESUMEN

1. *La falacia del extensionalismo.* Algunos lógicos han intentado simplificar sus sistemas diciendo que si las oraciones S_1 y S_2 tienen el mismo valor veritativo, entonces dos oraciones compuestas arbitrarias que sólo difieren en que una contiene S_1 donde la otra contiene S_2 deben tener el mismo valor veritativo también. Si esta "ley de extensionalidad" fuera cierta, y "x cree que el pasto es rosado" y "x cree que el pasto es púrpura" fueran compuestos genuinos con "el pasto es rosado" y "el pasto es púrpura" como componentes, entonces ambos compuestos tendrían que tener el mismo valor veritativo, puesto que los correspondientes componentes son ambos falsos. No siendo necesariamente así, concluyen que "x cree que el pasto es rosado" no es un compuesto genuino, con "el pasto es rosado" como componente; técnicamente, que no es una función genuina con "el pasto es rosado" como argumento. Mi conclusión, en cambio, es que la ley de la extensionalidad es falsa, y sólo vale en una área específica de la teoría lógica.

Suele argumentarse que si esta ley es falsa entonces debemos admitir que algunas proposiciones no son verdaderas ni falsas. El argumento es el siguiente: (a) Si todas las proposiciones son verdaderas o falsas, sólo puede haber cuatro funciones proposicionales para cada argumento proposicional; (b) para cada una de estas funciones se cumple que si p y q tienen el mismo valor veritativo, entonces esa función de p tendrá el mismo valor veritativo que esa función de q; por lo tanto, (c) si alguna función no obedece a la ley de extensionalidad no puede ser ninguna de las cuatro mencionadas, y si hay otras funciones además de ellas, entonces deben haber más posibles valores veritativos que las generen.

Pero el argumento queda invalidado al observarse que presupone lo que pretende probar: que el único rasgo de p del cual puede depender el valor veritativo de cualquier función de p es el propio valor veritativo de p; pues la lista de posibles funciones no incluye ninguna que, por ejemplo, sea verdadero para algunos argumentos verdaderos y falsa para otros argumentos verdaderos, como es el caso de "x cree que p". ¿Pero por qué diablos el valor veritativo de una función de p no puede depender de algún rasgo de p que no sea su valor veritativo? Que se dé o no el caso de que x cree que p no depende de si se da o no el caso de que p, sino de si p es creído o no por x.

2. *La teoría de Frege sobre funciones y valores.* El villano de esta obra es quizás Gottlob Frege; y el origen de la confusión se halla quizás en su invención del término "truth-value" (valor veritativo). Según él las oraciones denotan objetos llamados Verdad y Falsedad, del mismo modo que los numerales, y las fórmulas que los contienen, denotan números. Y así como el número denotado por una expresión funcional numérica sólo depende del número denotado por su argumento, el valor veritativo denotado por una función oracional sólo debe depender —según Frege— del valor veritativo denotado por la oración que le sirve de argumento.

Pero el paralelismo establecido por Frege no es válido. La verdad y la falsedad se asemejan más a propiedades de lo denotado por las oraciones que a las cosas mismas denotadas por ellas. Y, ciertamente, no son la denotación de las oraciones; pues en rigor las oraciones no denotan nada, y las proposiciones son cosas-con-propiedades sólo en un sentido Pickwickiano. Pero podemos formular este sentido de una manera precisa, y usar la locución sobre cosas-con-propiedades de un modo innocuo. Lo mismo vale para los números, que tampoco son cosas-con-propiedades. Por ejemplo, si decimos que 1 es mayor que 0, lo que queremos decir es que para cualquiera ψ y φ, si exactamente una cosa es φ y ninguna es ψ, entonces hay más cosas que son φ que cosas que son ψ. Y hay innumerables cosas que podemos "decir acerca de las proposiciones", en el sentido en el que podemos decir acerca de ellas que son verdaderas o falsas, así como hay innumerables cosas que podemos "decir acerca de los números" en el sentido en que podemos decir acerca de ellos que son o no mayores que 0.

Frege mismo sabía que hay más cosas que decir acerca del funcionamiento de las oraciones, además de que denotan la Verdad o la Falsedad, pues hablaba también del "sentido" de las oraciones, y reconocía la existencia de funciones genuinas del sentido de las oraciones. Por ejemplo, "X cree que el pasto es rosado" sería una función del sentido de "el pasto es rosado". Pero esto es introducir una dicotomía donde no hay ninguna. Las funciones veritativas y las funciones de creencia son funciones de los mismos argumentos.

Debemos agregar, sin embargo, que aunque es equívoco hablar de las oraciones como denotando valores de verdad en el sentido en que los nombres propios denotan individuos, es muy iluminador decir que los valores veritativos son la "extensión" de las oraciones, así como las clases son la extensión de los predicados. Trazaremos este paralelo luego que hayamos dicho algo sobre la identidad proposicional, pero podemos adelantar que parte de este paralelo consiste en el hecho de que ni los valores veritativos ni las clases son objetos genuinos, sino sólo "construcciones lógicas", y muy similares.

3. *Equivalencia e identidad proposicional.* El extensionalismo puro (no mitigado por la distinción fregeana entre sentido y denotación) iguala la identidad de lo que las oraciones significan con la identidad de sus valores veritativos, es decir, con su "equivalencia material". Pues lo que los extensionalistas dicen acerca de la equivalencia material: que todas las funciones de oraciones materialmente equivalentes son materialmente equivalentes, es realmente cierto de la identidad de significado, o, en otros términos, de la identidad de las proposiciones que las oraciones expresan. Si la proposición de que p es realmente la misma que la proposición de que q, entonces cualquier función de la proposición de que p es la misma proposición que esa función de la proposición de que q. Por ej., si la proposición de que *todos los solteros son no casados* es realmente la misma que la proposición de que *todos los no casados son no casados,* entonces la proposición de que *Juan se pregunta si todos los solteros son no casados* es la misma proposición que *Juan se pregunta si todos los no casados son no casados.*

Puede objetarse que hablar de identidad proposicional nos obliga a abandonar la idea de que las proposiciones son construcciones lógicas y a tratarlas como objetos genuinos. Como la cuantificación, la identificación implicaría un 'compromiso ontológico' no deseado, en este caso con proposiciones. Pero no es así.

Escribimos "Ipq" en lugar de "La proposición de que p es la misma que la proposición de que q". Los nombres aparentes "La proposición de que p" y "la proposición de que q" no figuran en el complejo *Ipq;* en la versión verbal, estos nombres aparentes pueden ser considerados como careciendo de significado o función fuera del complejo

1) "La proposición de que —es la misma que la proposición de que—", que abreviamos con "I— —", y donde los blancos están en lugar de oraciones y no de nombres. La forma guarda una exacta analogía con

(2) "La proposición de que —implica la proposición de que—" que es sólo un modo de escribir

(3) "si —entonces—",
donde tales nombres aparentes no figuran. La única diferencia consiste en que no disponemos de una forma conversacional análoga a (3) que nos permita traducir (1) pero ello es sólo un accidente lingüístico.

Las leyes principales de identidad proporcional son *Ipp,* y la mencionada antes, según la cual todas las funciones de proposiciones idénticas son idénticas. Escribiendo C α β en lugar de "Si α

entonces β" y usando d como una variable de expresiones que forman una oración a partir de otras, podemos formular esta segunda ley como *CIpqIdpdq.*

Puede objetarse esta ley sobre la base de que es plausible decir que la proposición de que todos los solteros son no casados es la misma que la proposición de que *todos los no casados son no casados,* y sin embargo que la proposición de que *Juan se pregunta si todos los solteros son no casados* no es la misma que la proposición de que *Juan se pregunta si todos los no casados son no casados.* Este tipo de objeciones dependen, sin embargo, de una confusión entre preguntarse si todos los solteros son no casados, y preguntarse si lo expresado por la oración "Todos los solteros son no casados" es verdadero. Y por 'preguntarse si lo que es expresado por la oración "Todos los solteros son no casados" es verdadero', *no* quiero decir lo mismo que preguntarse, con respecto a lo que es expresado por la oración "Todos los solteros son no casados", si es verdadero; pues esto es lo mismo que preguntarse si todos los solteros son no casados. Estoy refiriéndome, en cambio, al hecho de preguntarse, con respecto a la oración "Todos los solteros son no casados", si lo que expresa es verdadero. Y alguien puede preguntarse esto *sin* preguntarse, con respecto a la oración "Todos los no casados son no casados", si lo que expresa es verdadero. Pues alguien podría no saber que la oración "Todos los solteros son no casados" significa simplemente que todos los no casados son no casados.

4. *La objeción referente a las expresiones con comillas.* Si en la fórmula *CIpqIdpdq,* la variable d representa *cualquier* expresión que forma oraciones a partir de oraciones, entonces es posible presentar una objeción más sustancial a la mencionada ley. Pues un modo de formar una oración a partir de la oración "Todos los solteros son no casados" es encerrarla entre comillas y prefijar al resultado de la expresión "Juan pronunció la oración"; o sea que "Juan pronunció la oración '- - -' " es una expresión que forma oraciones a partir de oraciones, entendiéndose que las comillas internas son parte de esa expresión misma. Pero es evidente que la proposición de que Juan pronuncia la oración "Todos los solteros son no casados" *no* es la misma que la proposición que Juan pronunció la oración "Todos los no casados son no casados".

No se evita esta objeción diciendo, por ejemplo, que *CIpqIdpdq* sólo vale cuando la expresión representada por d no termina con comillas. Pues (a) hay expresiones que no terminan con comillas y a las cuales la ley no es aplicable; y (b) hay expresiones que terminan del modo indicado y que, sin embargo, se ajustan a la ley.

Para salvar la ley de identidad proposicional se requiere otro tipo de consideraciones. Aún como parte de un cálculo formalizado, la fórmula *CIpqCdpdq*, al igual que cualquier otra fórmula que expresa una ley lógica, puede usarse para *decir* algo. Más exactamente, lo que puede usarse así es su "clausura" o sea

(1) $\pi p \pi q \pi d$ *CIpqCdpdq*,

o en semi español:

(2) Para toda *p*, para toda *q* para toda *d*, si la proposición de que *p* es la misma que la proposición de que *q*, entonces si *dp* entonces *dq*.

Y estas leyes *no* se usan en lógica para decir algo *acerca* de oraciones o expresiones. (2) puede ser ejemplificada parcialmente por

(3) Para toda *p*, para toda *q*, si la proposición de que *p* es la misma que la proposición que de *q*, entonces si alguien resuelve que *p* entonces alguien resuelve que *q*, lo cual, a su vez, puede ser ejemplificado por:

(4) Si la proposición de que a todos los solteros se les aplica una multa de 20 libras es la misma que la proposición de que a todos los no casados se les aplica una multa de 20 libras, entonces si alguien resuelve que a todos los solteros se les aplica una multa de 20 libras, entonces alguien resuelve que a todos los no casados se les aplica una multa de 20 libras.

Lo que se quiere decir con (1) y (2) sólo tiene este tipo de ejemplificaciones. No contamos como tales los casos en los que las oraciones más largas *no* son acerca de lo mismo que las oraciones más pequeñas, sino acerca de estas oraciones mismas; la razón es que lo significado por (1) y (2) es algo que *no posee* tales ejemplificaciones.

5. *Valores veritativos y clases.* Diremos primero algo sobre de lo que realmente significa hablar acerca de clases. La forma más elemental de un enunciado que habla ostensiblemente acerca de clases es la de un enunciado que dice que algo es un miembro de ella, o sea un enunciado de la forma "x es un φ" (x is a φ *-er*) o "x es un miembro de la clase de los φ" (*x is a member of the class of φ-ers*). Pero este enunciado es sólo ostensiblemente acerca de una clase, pues equivale a"x es φ" (x *φ's*). Y el enunciado "La clase de los φ está incluida en la clase de los ψ" significa simplemente "Toda lo que es φ es ψ" (*Whatever φ's ψ's*).

La cuestión se complica cuando deseamos contar clases y propiedades. Decir que exactamente un individuo es φ, es decir, que para algún x, *x es φ*, y para x e y cualesquiera, si x es φ e y

es φ, entonces x es el mismo individuo que y. Y el enunciado: "Prior y Quine coinciden en exactamente una cosa" se transforma en:

> Para algún p, Prior y Quine creen que p, y para p y q cualesquiera, si Prior y Quine creen que p, y Prior y Quine creen que q, entonces la proposición de que p, es la misma que la proposición de que q,

donde la última cláusula debe entenderse del modo indicado en la última sección. Y la forma: "La propiedad de ser φ es la misma que la propiedad de ser ψ" (*The property of φ-ing is the same as...*), o, más brevemente: "Ser φ es lo mismo que ser ψ (*To φ is the same thing as to ψ*), puede interpretarse como construída a partir de las expresiones "ser φ" y "ser ψ" mediante un functor binario del mismo tipo que "Todo lo que es ( ) es ( )". El componente "La propiedad de ser ( )" no debe entenderse como formando un nombre a partir de la expresión que va dentro de los paréntesis sino como parte inseparable de todo el functor, de modo que el uso de esta forma no nos compromete a aceptar la existencia de propiedades. Entendido esto, el enunciado: "Hay exactamente una propiedad que no se aplica a nada" se transforma en: Para algún φ, nada es φ, y, para φ y ψ cualesquiera, si nada es φ y nada es ψ, entonces ser φ es la misma cosa que ser ψ.

Por supuesto esto es falso: ser una sirena no es lo mismo que ser un centauro. Pero, por otra parte, la *clase* de las sirenas es la misma que la de los centauros. Es conveniente hablar como si las clases fueran entidades, las cuales son idénticas cuando los predicados que las definen se aplican a los mismos objetos, o sea cuando todo lo que es φ es ψ y todo lo que es ψ es φ. Los predicados binarios pueden ser asociados análogamente con "relaciones en extensión". Y cuando φ y ψ son "predicados de grado cero", o sea proposiciones completas, podemos decir que sus extensiones son idénticas cuando se cumple que φ es formalmente equivalente a ψ. Y esto ocurre cuando y sólo cuando o bien se dan tanto φ como ψ, o bien no se da φ ni ψ. De modo que acuñamos el término "valor veritativo" para lo que describimos como idéntico cuando se cumplen las condiciones mencionadas. Pero debemos recordar, en todos los casos, que los supuestos sobre identidad no significan ni más ni menos que el enunciado de tales condiciones.

Estos usos extraños de la expresión "el mismo", y la invención de entidades a las que se apliquen, son altamente convenientes desde el punto de vista simbólico, pero no comprender adecuadamente el funcionamiento de estos artificios es filosóficamente desastroso. Por ejemplo, está de moda considerar que no tenemos derecho a hablar de la identidad de esto o aquello a menos que podamos formular las "condiciones de identidad" de las entidades en cuestión. Esta exigencia resulta razonable cuando estamos embarcados, como antes, en procedimientos de abstracción extensional, o sea cuando *realmente* no está en juego en absoluto la identidad, y cuando por lo tanto es necesario explicar qué sustituto de ella estamos empleando; pero en la mayor parte de los casos esta exigencia carece de sentido (como la cuantificación, la identidad es lo que es y no otra cosa). El error al que se dedica principalmente este artículo, o sea el tratamiento de los valores veritativos como los *denotata* de las oraciones y la confusión entre identidad proposicional y equivalencia material, es un caso especial de este error más general.