DIFFERENTIAL AND PROJECTIBLE PREDICATES*

LAURENCE FOSTER
University of Massachusetts

Another attempt has been made to solve Goodman's New
Riddle of Induction without appealing to the entrenchment of
predicates. In a recent article ('Some Thoughts on Good-
man's Riddle', *Analysis*, March, 1967) Michael Anthony
Slote proposes what he calls a 'partial answer' to Goodman's
Riddle, believing it to be epistemologically deeper than
Goodman's own solution. What I wish to show here is that
his proposed solution is completely inadequate.

In order to solve Goodman's Riddle, Slote introduces a
twofold distinction — one is between differential and non-
differential predicates and the other is between inductively
vicious and inductively non-vicious predicates.[1] Slote does
not explicitly state how these distinctions give us a partial
solution to Goodman's Riddle. We shall assume that Slote
intends to be giving either a set of necessary or sufficient
conditions for a predicate to be projectible. The specific point
of this paper is to show that even if Slote's distinctions are
accepted as legitimate (and no Goodmanean would accept
them as legitimate) they are irrelevant to the problems of
projectibility. First, it will be shown that being a non-dif-
ferential predicate is neither a necessary nor sufficient con-
dition for being non-projectible. Then it will be shown that
being an inductively vicious predicate is neither a necessary
nor sufficient condition for being non-projectible.

[1] Slote speaks indifferently of predicates or properties. For the sake of
convenience I shall follow him on this matter.

Slote defines a *differential property* or *characteristic* in the following way: 'A characteristic f is differential if and only if, given that anything X has f and anything Y lacks f, it follows logically (it is logically impossible for it not be the case) that X and Y are not exactly (or entirely) alike (or similar).' (p. 128) Leaving aside the obvious problems with this definition (i.e., the problems of what it is for two objects not to be exactly alike as well as the problem of what sense of 'logically' is being employed here) we can help to clarify our intuitions on the distinction between differential and non-differential predicates by relying on Slote's examples. As it turns out Slote claims that 'green' is a differential predicate while 'grue' is a non-differential one. This is so, Slote argues, since from the fact that one thing is green and the other not green, it follows logically that they are not exactly alike. However, the same does not hold for the predicate 'grue'. It does not follow logically from the fact that one object is grue and the other not grue that they are not exactly alike. For example, two emeralds may be exactly alike although one is green and examined before t, and the other green but not examined before t. Hence, one would be grue and the other not grue, yet they might be exactly alike.

Given this distinction one would expect Slote to argue that all non-differential predicates are non-projectible, and in this way give us a criterion for non-projectibility which does not rely on entrenchment. Oddly enough, Slote does not accept this idea, for he cites an example of a non-differential predicate which is projectible; namely, the predicate 'is formed in igneous rock'. Additional non-differential yet projectible predicates could easily be found; for example, any spatial predicate like 'arctic' or 'terrestrial' presumably would be non-differential according to Slote. Such predicates, however, are clearly projectible. This shows that being a non-differential predicate is not a sufficient condition for being non-projectible.

Perhaps, then, Slote's claim is not that all non-differen-

102

tial predicates.are non-projectible, but rather that all non-projectible predicates are non-differential. But if this is Slote's claim then it is also false. Consider the predicate 'emerub' which we can define as follows:

$$\text{emerub } x =_{Df} \text{ emerald } x \text{ or ruby } x[1]$$

Given Slote's definition of a differential property, it is clear that if any predicate is differential, the predicate 'emerub' is. If one object is an emerub (i.e. an emerald or a ruby) and another is not an emerub, it follows logically (in whatever sense of 'logically' that Slote has in mind) that the two objects are not exactly alike. Nevertheless, the predicate 'emerub' is intuitively non-projectible and it can be shown to be non-projectible by the following example. Suppose we have examined some, but not all, emeralds for color and have found that they are green. Also, suppose that we have not examined any rubies for color. Then, according to the second of Goodman's rules (p. 103) the hypothesis
   H₁ All emerubs are green
is eliminated in favor of the conflicting hypothesis
   H₂ All emeralds are green.
The predicate 'emerub' is clearly non-projectible in such a situation, and, as desired, it turns out to be non-projectible because of its elimination by the second of Goodman's rules. But the predicate 'emerub' is differential. This shows that being a non-differential predicate is not a necessary condition for being non-projectible.

Clearly, we could construct at will predicates like 'emerub' which are non-projectible yet are differential according to Slote's definition. But if some reader is still not convinced that Slote's proposed solution to Goodman's Riddle is com-

[1] For a similar example see J. Hullett, and R. Schwartz "Grue: Some Remarks", *The Journal of Philosophy*, LXIV, 1967, p. 269, and also Goodman's predicate 'bagleet' in *Fact, Fiction, and Forecast*, Indianapolis: Bobbs-Merrill, 1965, p. 111. All future references to *Fact, Fiction, and Forecast* are to this edition.

pletely unsatisfactory we may consider yet another example which relies on one of Slote's favorite properties.

Let us assume that we have examined some emeralds, say, a, b, c, and d for color and have found each of them to be green. Also, let us introduce the predicate 'gree' which we define in the following way:

x is gree $=_{Df}$ x is exactly like a or b or c
or d and green or is not exactly
like a or b or c or d and is not
green[1]

The predicate 'gree' is true of each of the four emeralds a, b, c, and d since each one of the emeralds is presumably exactly like itself. But the discovery that each of the four emeralds is gree does not increase the credibility of the hypothesis.

H_3 All emeralds are gree

We can show that this hypothesis is not projectible for it conflicts with H_2, and it is eliminated by the first of Goodman's rules. In such a situation the predicate 'gree' is nonprojectible. But it should be apparent that if any predicate is differential then surely 'gree' is. Given that one object is gree while the other is not gree wouldn't it follow logically (in Slote's sense of 'logically') that the two objects are not exactly alike? Once again, we find that being a non- differential predicate is not a necessary condition for being non-projectible.

It should be obvious from the preceding remarks that even if the alleged distinction between differential and non-dif-

---

[1] If the reader is unhappy with this predicate because it relies on the notion of exact likeness between different objects then he should be equally unhappy with Slote's definition of a differential property. In any case, a similar predicate can easily be defined which raises the same problem. Let the predicate 'greeb' be defined as applying to any object which is either identical with a or b or c or d and green, or not identical with a or b or c or d and not green. Cf. I. Scheffler, *The Anatomy of Inquiry*, New York: Alfred A. Knopf, 1963, p. 308.

ferential predicates were legitimate, it would completely fail to mark the difference between projectible and non-projectible predicates. Being a non-differential predicate is neither a necessary nor sufficient condition for being non-projectible. Hence, the distinction Slote draws between differential and non-differential predicates is not one which can be used to replace or supplement Goodman's solution to the New Riddle.

As mentioned above, Slote introduces a second distinction between inductively vicious and inductively non-vicious predicates in order to show how the distinction between diferential and non-differential predicates is relevant to the problems of projectibility. This new distinction, however, can be shown to be no more helpful than the previous one.

Slote explains the nature of an inductively vicious property as follows: 'F is an inductively vicious property if and only if there is a present or future time t such that to claim that all X's have f on the basis of a sample of X's examined (if examined at all) only after t will not be entirely like any of the X's one has already (i.e., at the time one is making the generalization in question) examined'. (p. 130) This explanation is fairly unintelligible as it stands since it lacks a subject term for the predicate 'will not be entirely like'. Perhaps, this is due to a misprint. Fortunately, we can see what Slote has in mind by considering the following example. According to Slote,

> '...grueness is inductively vicious because when one generalizes from the fact that all emeralds examined so far have been grue to the conclusion that all emeralds are grue, one is in effect saying that there is some present or future time t (depending on how t is specified in the definition of 'grue') such that any emerald not examined before t will (to some degree) be unlike the emeralds that have already (i.e., at the time one is generalizing) been examined.' (p. 130)

105

It should be apparent that the predicate 'green' is inductively vicious if 'grue' is — but to argue this point would be beyond the scope of this paper and would only be necessary if somehow the distinction between an inductively vicious predicate and an inductively non-vicious one turned out to be relevant to the problems of projectibility. But it is clear that this distinction is irrelevant. We have previously seen that the predicates 'emerub' and 'gree' are non-projectible. But, if the predicate 'green' is inductively non-vicious then, surely, so are the predicates 'emerub' and 'gree'. To use Slote's terminology we may say that if some unexamined green objects may be exactly like some examined green objects then surely some unexamined emerubs may be exactly like some examined emerubs. The same holds for gree objects (but presumably not for greeb objects). In addition, the time element in the notion of an inductively vicious predicate is absent in the case of the predicates 'emerub' and 'gree'. These predicates make no explicit reference to a time and make no more of an implicit reference to a time than do any other predicates.

It is worth noting that the same even holds for the grue-like predicates; for we may define the predicate 'grone' as applying to anything which is either green and located anywhere except Bayonne, New Jersey, or blue and located in Bayonne, New Jersey. (Unlike the predicate 'emerub' and 'gree', the predicate 'grone' presumably would be non-differential according to Slote). If we now suppose that we have not examined any emeralds for color in Bayonne, New Jersey but have examined many emeralds for color in other parts of the world and have found them all to be green then the hypothesis.

H₅ All emeralds are grone

although supported, non-violated, and unexhausted is eliminated by the first of Goodman's rules because of its conflict with H₂. H₅, then, is non-projectible. The predicate 'grone' is a non projectible inductively non-vicious predicate. Accord-

ingly, some non-projectible non-differential predicates are inductively non-vicious, and some non-projectible differential predicates (e.g., 'emerub' and 'gree') are also inductively-non-vicious. Being inductively vicious is not a necessary condition for being non-projectible.

As a last resort Slote may wish to maintain that being inductively vicious is a sufficient condition for non-projectibility. But this also is false. Consider the following sequence of numbers: 1, 3, 5, 7, 9, 11, 13, 15..., (If desired, the reader may give an interpretation to these numbers so that they are taken as recording the results of some scientific experiment. One could also, if desired, make the time element explicit in this example). Suppose we are asked to project the next number in the series. Presumably, we would consider the number 17 as the one we ought to project. This is due to the fact that since every examined k^{th} member of the series, is equal to 2k-1 we are led to expect that every k^{th} member of the series, and, hence, the next number in the series is equal to 2k-1. But would Slote not have to regard us as projecting an unlikeness? Are we not in effect saying that at any future time any new k^{th} member of the series which is found will be unlike the previous k^{th} members? For example, the number 17 is not exactly like the previous members of the series, yet, it and every previous k^{th} member has the property of being equal to 2k-1. This clearly shows that some projections of unlikenesses into the future are projectible. It follows, then, that being inductively vicious is not a sufficient condition for being non-projectible. Even if the predicate 'grue' is inductively vicious while the predicate 'green' is not, this would be irrelevant to the question of which predicate is projectible.[1]

[1] It is worth noting here that one of Slote's basic problems stems from his belief that the grue-hypothesis projects an unlikeness into the future while the green-hypothesis projects a likeness. Here Slote is either presupposing the facts of entrenchment or else relying on some notion of likeness and unlikeness which is independent of language. But it should be clear to anyone who has read *Fact, Fiction, and Forecast* that whether or not a likeness or unlikeness is pro-

The proceding remarks should amply show that even if we accept Slote's distinctions they are irrelevant to the solution of Goodman's Riddle.

jected into the future is dependent upon the language we use to make our projections. Taking off on a remark from Goodman we may say that likenesses are where you find them, and you can find them anywhere. The same can be said for unlikenesses. It is hopeless then to try to distinguish between pro- jectible and non-projectible predicates solely in terms of the projection of likenesses rather than of unlikenesses.

En un artículo reciente ("Some Thoughts on Goodman's Riddle", *Analysis,* Marzo 1967), Michael Anthony Slote propone una "respuesta parcial" al nuevo enigma de la inducción planteado por Goodman. Lo que quiero mostrar en este trabajo es que la solución propuesta por Slote es completamente inadecuada.

Slote introduce una doble distinción: una entre predicados diferenciales y predicados no-diferenciales y la otra entre predicados inductivamente viciosos y predicados inductivamente no-viciosos. Dado que no es claro el propósito de Slote al introducir estas distinciones, asumamos que lo que quiere hacer con ellas es señalar un conjunto de condiciones necesarias o suficientes para que un predicado sea proyectible. Veremos que aunque se acepten como legítimas las distinciones de Slote, resultan irrelevantes para resolver los problemas de proyectibilidad. Mostraré que ser un predicado no-diferencial no es una condición necesaria ni suficiente para ser un predicado no-proyectible. Luego trataré de probar que ser un predicado inductivamente vicioso no es tampoco una condición necesaria ni suficiente para que un predicado sea no-proyectible.

Slote define una *característica o propiedad diferencial* de la siguiente manera: "Una característica f es diferencial, si y sólo si, dado que toda X tenga f y toda Y carezca de f, se sigue lógicamente (es lógicamente imposible que no sea así) que X y Y no sean exactamente (o enteramente) iguales (o similares)" (p. 128). De acuerdo con Slote "verde" sería un predicado diferencial.

Dada esta distinción podría uno esperar que Slote argumentara que todos los predicados no-diferenciales son no-proyectibles, pero Slote no acepta esta idea, ya que da ejemplos de predicados que son no-diferenciales pero proyectibles. Ser un predicado no-diferencial no es pues una condición suficiente para ser un predicado no-proyectible.

Probablemente lo que Slote quiere afirmar es que todos los predicados no-proyectibles son no-diferenciales. Pero esto es también falso. Consideremos el predicado "esmerub" el cual puede definirse así: $\text{esmerub}_x = \text{Df. esmeralda}_x$ o rubí_x. El predicado "esmerub" es pues diferencial. Sin embargo, "esmerub" es intuitivamente no-proyectible y que esto es así puede mostrarse mediante el siguiente ejemplo: Supongamos que se han examinado algunas, no todas las esmeraldas, con el objeto de determinar qué color tienen y se ha encontrado que todas son verdes. Supongamos también que no se ha

examinado ningún rubí para determinar su color. Entonces, de acuerdo con la segunda regla de Goodman (p. 103), la hipótesis:

H₁ Todos los esmerubs son verdes,

se elimina en favor de la hipótesis con la que entra en conflicto:

H₂ Todas las esmeraldas son verdes.

El predicado "esmerub" es claramente no-proyectible en tal situación y sin embargo es diferencial. Esto muestra que ser un predicado no-diferencial no es una condición necesaria para ser un predicado no-proyectible. Por lo tanto, la distinción entre predicados diferenciales y no-diferenciales no puede ser usada para reemplazar o suplementar la solución de Goodman al nuevo enigma de la inducción.

La segunda distinción introducida por Slote entre predicados inductivamente viciosos e inductivamente no viciosos, no ayuda tampoco a la solución del problema.

Una propiedad inductivamente viciosa es caracterizada por Slote de la siguiente manera: "F es una propiedad inductivamente viciosa, si y sólo si hay un tiempo t presente o futuro tal que, decir que todas las X tienen f, sobre la base de una muestra de X examinada después de t (en el caso en que sea eximanada), no será enteramente igual a todas las otras X que ya (en el momento en que se hace la generalización en cuestión) se han examinado" (p. 130). Esta explicación resulta ininteligible, pues no aparece ningún término sujeto para el predicado "no será enteramente igual". De cualquier forma la distinción entre predicados inductivamente viciosos y predicados inductivamente no viciosos resulta irrelevante para la solución a los problemas de proyectibilidad. Hemos visto que el predicado "esmerub" es no-proyectible. Pero si el predicado "verde" es inductivamente no vicioso, también lo será el predicado "esmerub". Podríamos decir que si algunos objetos verdes aún no examinados pueden ser exactamente como algunos objetos verdes ya examinados, entonces también algunos esmerubs no examinados pueden ser exactamente como algunos esmerubs ya examinados. El elemento tiempo que forma parte de la noción de predicado inductivamente vicioso, está ausente en el caso del predicado "esmerub". Algunos predicados son pues no-proyectibles y a la vez inductivamente no viciosos. Podríamos añadir que algunos predicados no-proyectibles y no-diferenciales son inductivamente no viciosos y que algunos predicados no-proyectibles y diferenciales (por ejemplo "esmerub") son también inductivamente no viciosos. Por lo tanto, ser inductivamente vicioso no es una condición necesaria para ser predicado no-proyectible.

Como último recurso, Slote podría mantener que ser inductivamente vicioso es una condición suficiente para la no proyectibilidad.

110

Pero esto también es falso. Consideremos la siguiente secuencia de números: 1,3,5,7,9,11,13,15,..... Supongamos que se nos pide proyectar el siguiente número en la serie. El número 17 sería el que escogeríamos, debido al hecho de que cada K^o miembro examinado de la serie es igual a 2K-1. ¿Pero acaso no interpretaría esto Slote como proyectar una desigualdad? ¿No estamos acaso diciendo que en cualquier tiempo futuro, cualquier nuevo miembro K^o de la serie será distinto a los miembros K^{os} anteriores? Esto muestra claramente que algunas proyecciones de desigualdades en el futuro son proyectibles. De esto se sigue, pues, que el ser inductivamente vicioso no es una condición suficiente para ser no-proyectible.

Las consideraciones anteriores deben mostrar con amplitud que, incluso en caso de aceptar las distinciones introducidas por Slote, resultan irrelevantes para resolver el enigma planteado por Goodman.