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SUBJUNTIVES

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1. In *Reference and Generality*[1] Peter Geach contrasts

> Just one man broke the bank at Monte Carlo, and he has recently died a pauper,

with

> Smith broke the bank at Monte Carlo, and he has recently died a pauper,

and says that in the former it is impossible to find any noun or noun phrase for which 'he' goes proxy. Clearly however it is replaceable with 'that man', as Geach admits, so the difficulty is giving a formal account of such an anaphoric phrase. But that is a difficulty only in Fregean logic, since in Hilbert's ϵ-calculus, as I showed in "E-type Pronouns and ϵ-terms"[2] we can symbolise the former

$$(\exists x)(Mx \cdot Bmx \cdot (y)((My \cdot Bmy) \supset y = x)) \cdot D\epsilon x(Mx \cdot Bmx \cdot (y)((My \cdot Bmy) \supset y = x)),$$

and the epsilon term expressly does the job of reference which 'he' and 'that man' do, in natural speech. Because of the lack of this facility in Fregean logic Geach wants

[1] Cornell, 1962, p. 125.
[2] *Canadian Journal of Philosophy*, 1986.

to parse the whole sentence not as a conjunction, but as a quantified conjunction, viz

$$(\exists x)(Mx \cdot Bmx \cdot (y)((My \cdot Bmy) \supset y = x) \cdot Dx)$$

and the two forms are indeed equivalent, but since basic Fregean logic lacks the epsilon term the exact thing it cannot do is revealed, namely extract a pronoun from out of a bound expression.

Now, Geach's attempt to treat all pronouns with quantified antecedents as bound by those antecedents is again evident in his treatment[3] of the 'donkey sentence'

If Smith owns a donkey, he beats it.

As is common in the Fregean tradition Geach identifies this conditional with the universal statement

Any donkey Smith owns he beats,

i.e. the quantified conditional

$$(x)((Dx \cdot Osx) \supset Bsx),$$

so he no more has a conditional representation for the conditional sentence than he had a conjunctive representation for the conjunctive sentence before. In fact, as Gareth Evans has shown[4] Fregean logic has no representation for Geach's conditional sentence, since it is distinct from the quantified conditional. In epsilon terms its formalization is

$$(\exists x)(Dx \cdot Osx) \supset Bs\epsilon x(Dx \cdot Osx),$$

[3] *Op. cit.*, p. 128.

[4] "Pronouns, Quantifiers, and Relative Clauses", *Canadian Journal of Philosophy*, 1977; see also, for instance, F. Sommers' *The Logic of Natural Language*, Oxford, 1982, Ch. 4, and J. Hintikka and L. Carlson's "Conditionals, Generic Quantifiers and Other Applications of Subgames" in E. Saarinen (ed.), *Game-Theoretical Semantics*, Reidel, 1978.

i.e.

$$(Da \cdot Osa) \supset Bsa,$$

where $a = \epsilon x(Dx \cdot Osx)$. This formalization shows that the conditional is derivable from the quantified conditional, but no vice versa.

But having thus isolated a (material) conditional distinct from the quantified conditional, questions remain about its interpretation and use. In the schematic case

$$(\exists x)Sx \supset P\epsilon xSx,$$

it is clear that only in extreme circumstances (if $(x)(Sx \supset Px)$, and if $(x)Sx \cdot (x) - Px$) does this conditional have a determinate truth value, since the epsilon term 'ϵxSx', in the consequent, in general has a choice of referents. But that does not mean that the conditional does not have a determinate ***probability***: indeed it is easy to see that the probability of this conditional is a conditional probability.

For, supposing $(\exists x)Sx$, we have, by the epsilon calculus, that $S\epsilon xSx$, and hence the choice of referent for 'ϵxSx' is limited to the set of S's. But then the probability of the conditional is the probability of $-S\epsilon xSx \vee P\epsilon xSx$, which is just the probability of $P\epsilon xSx$, and so is the probability of P given S, *i.e.* prob (P/S). This conditional probability is commonly left undefined in the reverse case, where $-(\exists x)Sx$, but if we arbitrarily stipulate that it is then 1 we get the unrestricted relation

$$\text{prob}((\exists x)Sx \supset P\epsilon xSx) = \text{prob}(P/S),$$

since, clearly, if $-(\exists x)Sx$, the left-hand-side is also 1.

This means, for one thing, that the conditional is what has sometimes been called an 'indefinite proposition',[5] *i.e.* that 'if anything is S it is P' may also be read 'an S is a P'. But the link with conditional probability may also put in mind recent theories of subjunctive conditionals, and so an expression for these seems to be near to hand. However, a more general matter to do with their symbolisation must first be clarified.

2. There has been a substantial tradition in recent philosophical logic which has questioned whether material implication, *i.e.* hook, satisfactorily captures the ordinary language particle 'if'. But those seeking to deny the truth-functional definition of 'if' have a hard job on their hands. Either they must deny the standard link with disjunction, or deny the truth functional definition of 'or'; and likewise for conjunction and 'and'. Most commonly the line would be that 'if' has many uses, and that, while hook captures the most basic of these, other senses of the term require different formulations.

For instance, it is sometimes said that another sense of 'if' is captured by fish-hook, *i.e.* 'strict implication' rather than plain material hook. We may define

Necessarily if p then q,

as

$$\mathrm{L}(p \supset q),$$

where necessity is defined in terms of 'truth in all possible worlds', (or better, to avoid 'Modal Realism': 'what would be true in all possible worlds'), and such an expression is commonly symbolised (indeed was originally symbolised) in a conditional 'fish-hook' form. But that doesn't show that 'if' is ambiguous. For the given expression is a quantified conditional, *i.e.* it is to be sym-

[5] See B. D. Ellis' *Basic Concepts of Measurement*, Cambridge, 1966, p. 168; also, for instance, J. R. Lucas' *The Concept of Probability*, Oxford, 1970, Ch. IV.

bolised

$$(i)(V(p,\omega_i) = 1 \supset V(q,\omega_i) = 1)$$

or, for short, as I shall put it,

$$(i)(Wip \supset Wiq),$$

which, like the quantified 'donkey' conditional before, does not have a natural reading in a straight 'if P then Q' form. Therefore it is not a conditional, and does not introduce any new form of 'if'. That is not to deny there are necessitated conditionals, or that they have properties different from conditionals; but it is crucial we resist them being called 'conditionals'—otherwise we shall lose sight of the basic particle 'if', and its truth-fuctional definition—and might even be moved to cook up other 'conditional' forms.

Indeed, it is common to try to represent *subjunctive conditionals* by means of some non-hook, but grammatically hook-type expression. As a result of this it comes to seem that 'if it were the case that p then it would be the case that q', is not of the hook-form '$P \supset Q$', since it is certainly not the indicative 'if p then q'. But we should consider more carefully whether this conditional is not still of the form '$P \supset Q$', and the 'if' in it not still the straight material connective. For the difficulty in formalising subjunctives might instead lie in getting hold of a formalization of 'it would be the case that' not in getting hold of a new formalization of 'if'. In fact I prefer to write the elementary subjunctive statement 'it would be the case (in circumstances i) that p' as above, *i.e.* as 'Wip', where 'i' is some 'possible world' (or, better, world-description), *i.e.* where $(i)Wip \equiv Lp$ (also $Wip \equiv -Wi-p$, and $Wi(p \cdot q) \equiv (Wip \cdot Wiq)$). One subjunctive conditional compound then has the expression '$Wap \supset Waq$' where 'a' makes reference to some set of circumstances elsewhere given ('*then*, had p been true, q would have been true'); another subjunctive conditional has the expression '$Wbp \supset Wbq$', where $b = \epsilon i Wip$, and there is no external reference ('supposing it ever were that p, *then* it would be that q'). Clearly the latter conditional may also be expressed

$$(\exists i)Wip \supset W(\epsilon i Wip)q$$

and we have derived from general principles the expression for a subjunctive conditional with many of the currently desired properties.[6]

Prime amongst these properties, of course, is the previous equation between the probability of this type of conditional and the corresponding conditional probability, but there are also many 'fallacies' to be noted. For instance, one cannot derive '$Wc - q \supset Wc - p$', where $c = \epsilon i Wi - q$, from '$Wbp \supset Wbq$' (the fallacy of Contraposition). Nor can one derive '$Wd(p \cdot r) \supset Wdq$', where $d = \epsilon i Wi(p \cdot r)$, from '$Wbp \supset Wbr$' (the fallacy of Strengthening the Antecedent), or derive '$Wbp \supset Wbr$' from '$Wbp \supset Wbq$' together with '$Weq \supset Wer$', where $e = \epsilon i Wiq$ (the fallacy of Transitivity).

On the other hand, we do have 'Conditional Excluded Middle' valid, since necessarily '$(Wip \supset Wiq) \vee (Wip \supset Wi - q)$' is true, for any i. Also, iff Mp then $(Wbp \supset Wbq) \supset -(Wbp \supset Wb - q)$—since '$Wbp$' is '$(\exists i)Wip$', *i.e.* '$Mp$'. But we avoid many of the difficulties about the link between this type of conditional and the indicative one. For instance, Stalnaker wants to say that the subjunctive conditional '$A > B$' (sic) implies '$A \supset B$', but he presumes thereby that the subjunctive and the indicative conditional have the same parts, trying, as

[6] See, for instance W. L. Harper, R. Stalnaker and G. Pearce (eds.), *Ifs*, Reidel, 1981.

we have seen, to incorporate the 'it would be the case that' not into the parts, but into the connective. Certainly the indicative can be represented in the form of a subjunctive, *i.e.* as '$Wop \supset Woq$', where o is the completely true world-description (for which $(p)(Wop\ p)$). But that merely points up the fact that the difference between the two conditionals does not lie anywhere near where Stalnaker (and Lewis *et al.*) locate it. It lies not in the connective but in the respective parts, making the two conditionals almost as distinct as '$Wip \supset Wiq$' and '$Wop \supset Woq$', and certainly no entailment between them—though if it is probable that a p-world is a q-world that makes it probable that, in *this* world, if p, q, since $\text{prob}(q/p) \approx 1$ does entail $\text{prob}(p \supset q) \approx 1$.

Lewis' difficulty in this area was his derivation of the subjunctive conditional from '$p \cdot q$'; but clearly only '$Wop \supset Woq$' follows from this, not '$Wbp \supset Wbq$': 'p' entails 'Mp' and hence 'Wbp', but 'q' only entails 'Weq', not 'Wbq'. Lewis also thought he had an advantage over Stalnaker through allowing 'might' conditionals which contradicted the 'would' conditionals, and we have something like that facility here. But '$Wbp \cdot Wb-q$' (*i.e.* 'In some circumstances it would be true that p, and there that $-q$') is not a conditional, for one thing, and though '$M(p \cdot -q)$' and '$M(p \cdot q)$' (*i.e.* 'In some circumstances it would be true that p and that $-q$/that p and that q') can be true together, it is not the case that '$Wbp \cdot Wb-q$' and '$Wbp \cdot Wbq$' can be true together, as is allowed in Lewis' analysis of the 'might' forms, so the present analysis of 'might', while distinct from that of 'would', is no threat to 'Conditional Excluded Middle'. Certainly, in connection with this principle, it is rare for either of '$Wbp \supset Wbq$' and '$Wbp \supset Wb-q$', to be *settleable as true*, since as we saw at the start of this paper, the epsilon term in them has, in general, no uncontestable reference to fix the truth value of the pronomial consequents. But the fact that there is no way of deciding what nationality Bizet and Verdi would be, if they were compatriots, does not stop some judgement being *true*; indeed if there was no truth to the matter there could be no probability judgement, since probability is probability of being true. The desire to settle the truth of subjunctives has manifested itself in the formulation of various 'closeness' conditions, but there is no place for them here, once it is recognized such conditionals are indeed 'indefinite propositions' which only rarely figure assertively outside probability judgements.

3. I conclude that a detailed inspection of higher predicate calculi and standard modal logic is all that is required to give a proper analysis of subjunctive speech. There is one caveat, however, which must be added to this: for the other type of subjunctive conditional to that most considered above, namely the counterfactual 'Had it then been that p it would have been that q', *i.e.* '$Wip \supset Wiq$', with externally given i, has no higher logic than the standard propositional one—supposing we keep the circumstances constant, *i.e.* continue the same story. For '$Wip \supset Wiq$' is the same as '$Wi(p \supset q)$', and, whenever '$p \supset q$' is tautologous, '$Wi(p \supset q)$', is tautologous too. Hence we must keep the separation between the two subjunctive forms very clearly in mind when examining their occurrence in actual speech. For the one without an external reference can only satisfy 'Stalnaker's Hypothesis',[7] *i.e.* have a probability which is a conditional probability, by having a probability which itself cannot be conditionalized—on pain

[7] *Op. cit.*, p. 11.

of Lewis' triviality results.[8] Hence it is an absolute, or *a priori* subjunctive, not a temporal or limited one, *i.e.* 'If it ever were that p, then it would be that q' does not presuppose any particular world is being talked about. But that does not make such subjunctive conditionals 'trivial',[9] even if it means[10] we do not have the principle

$$((Wbp \supset Wbq) \cdot (Weq \supset Wep) \cdot (Wbp \supset Wbr)) \supset (Weq \supset Wer).$$

For the resulting fact that these conditionals can have little place in argument leaves it open for such conditionals to originate, in their antecedents, the existential presuppositions for any further story-telling.[11]

[8] *Op. cit.*, pp. 132, 134.

[9] *Op. cit.*, p. 13.

[10] *Op. cit.*, p. 14.

[11] For further work on Hilbert's epsilon calculus, see my "Hilbertian Reference" (forthcoming in *Noûs*); "Hilbertian Tense Logic" (*Philosophia*, 1987); "Intensional Identities" (forthcoming in *Logique et Analyse*); "Prior and Cresswell on Indirect Speech" (forthcoming in the *Australasian Journal of Philosophy*); "Fictions" (*British Journal of Aesthetics*, 1987).

RESUMEN

En este artículo se intenta dar un análisis de los condicionales subjuntivos a partir de principios lógicos generales usando, de manera específica, una formalización de los pronombres que utiliza términos épsilon hilbertianos que se han estudiado en otro lugar. Esto permite aislar un condicional, cuya probabilidad es una probabilidad condicional, y se tiene así un condicional subjuntivo que satisface la hipótesis de Stalnaker, y evade los resultados de trivialidad de Lewis. Sin embargo, resulta que se dispone de una segunda forma subjuntiva sin estas propiedades y se propone, para esclarecer esto, una formalización general de los subjuntivos en la que se toma 'sería el caso que' como un operador cuasi-modal.