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EXCLUDING THE MIDDLE

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In a number of recent papers ("E-type Pronouns and ϵ-terms", *Canadian Journal of Philosophy*, 1986; "Fictions", *British Journal of Aesthetics*, 1987; "Hilbertian Tense Logic", *Philosophia*, 1987; "Hilbertian Reference", *Noûs*) I have shown how Hilbert's ϵ-calculus resolves several outstanding problems in Philosophical Logic: in the present paper I want to show how epsilon terms, and the principle of choice on which they are based, enable us to defend The Law of The Excluded Middle against common doubts and questions; in particular they help us to see how this law is not under threat from the presence of fictional statements and various other 'undecidable' statements, including subjunctive conditionals. In addition the approach shows the law is not under threat from *vagueness*, and a straightforward 'choice' resolution of the Sorites Paradox is thereby obtained. Many recent writers have in fact been quite near to this style of solution to these problems: I shall proceed, therefore, by commenting upon a collection of passages from the current literature.

1

Hilbert's epsilon terms are an alternative formalization of Skolem's "Decision Functions". For technical details of the latter see, for instance, P. S. Novikov's *Elements of Mathematical Logic* (trans. L. F. Boron, Oliver and Boyd, 1964, pp. 123–128); for the complete technical details of the former see A. C. Leisenring's *Mathematical Logic and Hilbert's ϵ-symbol* (Macdonald, 1969). An informal introduction to epsilon terms might develop them from the following theorems of standard predicate logic:

(Ey)((Ex)Fx ⊃ Fy)

(Ey)(Fy ⊃ (x)Fx),

with regard to which Irving Copi said (*Symbolic Logic*, Macmillan, 1973, p. 110):

> An intuitive explanation can be given by reference to the ancient Athenian general and statesman Aristedes, often called 'the just'. So outstanding was Aristedes for his rectitude that the Athenians had a saying:
>
> If anyone is just Aristedes is just.
>
> With respect to *any* attribute, there is always some individual y such that if anything has that attribute, y has it. That is what is asserted by the [first proposition above]. If we turn our attention not to the attribute of being just, but to its reverse, the attribute of being corruptible, then the sense of the Athenian saying is also expressible as:
>
> If Aristedes is corruptible, then everyone is corruptible.
>
> Again generalizing, we may observe that with respect to *any* attribute there is always some individual y such that if y has that attribute, everything has it.

Standard predicate logic, however, has no facility for constructing the general name for such paradigm individuals y, and this is what Hilbert's epsilon calculus expressly permits.

More precisely, if we take '(Ex)Fx' as 'FϵxFx' then we can develop an enriched predicate calculus from the propositional calculus together with the one axiom (scheme)

$Fy \supset F\epsilon xFx$,

where 'ϵxFx' is a term for all predicates 'F' in the language (and, to avoid a clash of bound variables, 'y' must be 'free for "x" ' in 'F', see Leisenring, *op. cit.*, p. 12). The quantifiers are then introduced by means of the definition above, which means, in particular, that ϵxFx is possibly not F since '$-(Ex)Fx$' is, in general, contingent. 'ϵxFx' thus refers to some chosen F so long as there are F's, but it has a quite indeterminate reference if there are no F's, since it is then picked arbitrarily from the real universe at large.

That we need this facility in the theory of reference has been recognized by a number of writers, notably Fred Sommers. In *The Logic of Natural Language* (Oxford, 1982, p. 57), Sommers has the following point to make, with regard to what he calls 'epistemic reference':

> If we acknowledge a sense of reference for 'some S' we drop the usual identification requirements; it is then but a small step to recognize a sense of genuine reference that does not require a reference to an S. I may, for example, credulously say that a ghost is making a noise in the attic and what I have said is false, but although I fail to refer to what I purport to refer to, my actual reference is not necessarily vacuous or unsuccessful. For having said that a ghost made a noise, I might be told that it (the thing to which I referred by the referring phrase 'a ghost') was not a ghost, but the upstairs maid or a cat. To be told this is to be told that *what I took to be a ghost* was not a ghost. Thus the lack of a referent, as described by the referring phrase, prevents reference to something of that description, but it is no bar to reference to something that was taken to be a thing of that description. Let us call the kind of reference in which I take something to be so and so, 'epistemic reference'.

There is, in this passage, the core of the idea, developed in the papers above that fictions are (badly-referred-to) real things (and hence that all propositional attitudes are about reality). But, more immediatly, the formal fact is that there is no contradiction in '$-G\epsilon x(Gx.Nx)$', i.e. 'that ghost which was noisy in the attic was not a ghost'. Indeed, as we have just seen, it is a central element in the mechanism of Hilbert's ϵ-calculus that '$-F\epsilon xFx$' be given a sense: it is equivalent to '$-(Ex)Fx$', i.e. 'there is no F'. Remembering Aristedes, the cat is 'what would be a ghost if anything was'—but in fact, of course, the cat is not a ghost.

Sommers' linguistic point has also been made by Keith Donnellan ("Reference and Definite Descriptions", *Philosophical Review*, 75, 1966, see Sommers, *op.cit.*, p. 217, also my "Talking about Something" in *Analysis*, 1963).. It has been explicitly denied by Saul Kripke ("Naming and Necessity" in D. Davidson and G. Harman (eds.), *Semantics of Natural Language*, Reidel, 1972, p. 254):

> It is a point made by Donnellan, that under certain circumstances a particular speaker may use a definite description to refer, not to the proper referent ... of that description but to something else which he wants to single out and which he thinks is the proper referent of the description, but which in fact isn't. So you may say 'The man over there with the champagne in his glass is happy', though he actually only has water in his glass. Now, even though there is no champagne in his glass, and there may be another man in the room who does not have champagne in his glass, the speaker *intended* to refer, or maybe, in some sense of 'refer', *did* refer, to the man he thought had the champagne in his glass. Nevertheless I'm just going to use the term 'referent of the description' to mean the object uniquely satisfying the conditions in the definite description. This is the sense in which it's been used in the logical tradition. So, if you have a description of the form ' the x such that φx', and there is exactly one x such that φx, that is the referent of the description.

But reference *is* intentional, and Kripke's 'logical tradition' is one which, for instance, cannot solve Berry's Paradox where the unavoidable point has to be that 'the least number not denotable in English by a noun phrase with less than 100 letters' does not denote the least number not denotable in English by a noun phrase with less than 100 letters (for a full discussion of the Hilbertian resolution of this paradox, see my "Hilbert and Paradoxes", which is forthcoming). Kripke recognized (*op. cit.*, p. 255) that not every phrase of the form 'the x such that φx' is used as a 'description' rather than a name: but in fact *none* are, they are all 'rigid designators' in Kripke's sense.

The reason why this is so is that 'the φ is F' contradicts 'The φ is not F' i.e. 'the φ' is a 'logical subject' (*cf.* Sommers, *op. cit.*, p. 27). But the major difficulty in seeing the facts of this matter lies in seeing what happens when there is no single φ, i.e. when the subject is a fiction, for then the definite description would not 'refer', in Kripke's sense. But there is no difficulty in holding on to classical logic, in this case, and, indeed, there is a quite general argument why we should do so. Thus Czeslaw Lejewski (in "Logic and Existence", see G. Iseminger (ed.), *Logic and Philosophy*, Appleton-Century-Crofts, 1968, p. 171) says:

> A remedy that might suggest itself to an unscrupulous mind would be to ban the use of empty noun-expressions and consider them as meaningless. Quine is right in not following this course. One may disagree as to the truth-value of the proposition 'Pegasus exists', but one would have to have attained an exceptionally high degree of sophistication to contend that the expression was meaningless. Quine does not think that empty noun-expressions are meaningless just because they do not designate anything. He allows the use of such words as 'Pegasus', 'Cerberus', 'centaur', etc., under certain restrictions, and tries to distinguish between logical laws which prove to be true for any noun-expressions, empty, or non-empty, and those which hold for non-empty noun-expressions only. It follows from his remarks that before we can safely use certain laws established by logic we have to find out whether the noun-expressions we may like to employ are empty or not. This, however, seems to be a purely empirical question... This state of affairs does not seem to be very satisfactory. The idea that some of our rules of

> inference should depend on empirical information, which may or may not be forthcoming, is so foreing to the character of logical enquiry that a thorough re-examination of... inferences may prove to be worth our while.

Lejewski goes on to show there is a way of removing empirical considerations from logic, and retaining the classical laws, by adopting an 'unrestricted' rather than 'restricted' interpretation of the quantifiers (what some would call a 'substitutional' rather than 'objectual' reading). Speaking of '(Ex)(Fx ∨ −Fx)' (16) and '(x)Fx ⊃ (Ex)Fx' (17) Lejewski can then say (*op. cit.*, p. 176):

> Under the unrestricted interpretation... (16) and (17) come out to be true irrespective of whether the universe is empty or non-empty. For (16) is implied by any component of type 'Fa∨−Fa', where 'a' stands for a noun-expression... In the case of (17) we argue as follows: if we assume that the antecedent of (17) is true then a proposition of type 'Fa', where 'a' stands for an empty noun-expression, must also be true in harmony with the unrestricted interpretation of the universal quantifier. Now any such proposition implies the proposition of type '(Ex)Fx', which again must be true. Thus in the establishing of the truth value of (16) and (17) the problem of whether the universe is empty or non-empty is altogether irrelevant, on condition, of course, that we adopt the unrestricted interpretation of the quantifiers.

Hence, as was said before, there is no difficulty in retaining classical logic, and in particular, therefore, there need be no doubt that 'the φ is F' contradicts 'the φ is not F', even when there is not just one φ. This means the difficulty in accommodating fictions lies elsewhere: it lies in determining *which* of such pairs *are true*, in any case. Jonathan Cohen put the point, as follows, in direct response to Lejewski (Iseminger, *op. cit.*, p. 184):

> The trouble is that if we try to interpret the standard predicate calculus along Lejewski's lines we run into paradoxes about predication in the empty universe. Take an empty noun-expression 'y'. On Lejewski's interpretation 'Fy∨−Fy' is a logical truth

whatever meaning we assign to 'F', and so is '−(Fy. − Fy)'. Of the two statements 'Fy' and '−Fy' one must be true and the other false. But how are we to tell which is true and which is false, since neither is deducible within the system? If 'exists' is put for 'F' there is no difficulty: 'Fy' is false and '−Fy' is true, because, *ex hypothesi* 'y' is empty. But suppose some other predicate-expression, like 'is winged', 'is unwinged', 'is hot', 'is cold', etc. is put for 'F'. There seems no conceivable reason for assigning one truth value to 'Fy' and the other to '−Fy', though perhaps, if it had not been for the theorems 'Fy ∨ −Fy', and '−(Fy. − Fy)', we might plausibly have said that both are equally true, because there can be no evidence against either, or that both are equally false because there can be no evidence in favour of either.

The classical difficulty with this case, from a Hilbertian perspective, is therefore that *evidence* is expected for what can only be a matter of *choice*, i.e. *will*: for if $y = \epsilon x \varphi x$, then the essential fact about '$\epsilon x \varphi x$', when $-\varphi\, \epsilon x\, \varphi x$, is that its reference is quite undetermined, and only arbitrary nomination can specify it. What would Pegasize, if anything did, i.e. y, is not determined by the quality φ, but that does not prevent us specifying y, and hence settling the truth value of 'Fy', for all 'F'.

It is ultimately a belief in Determinism, I think, but also a belief that 'fictions' are not part of real life, which hides the apprehension of the facts of this matter. Thus Sommers says (*op. cit.*, p. 314):

To avoid the application of the sentential law of excluded middle which would require him to say that either the present King of France is bald or he isn't bald, Strawson proposes and argues for a truth-value gap for vacuous propositions. The semantic doctrine of existential presupposition is brought in to justify the truth-value gap; it serves the same purpose that Dummett's doctrine of the tie between being effectively decidable and having a truth value serves in connection with the statement that Jones was brave or he was not. Dummett's (other) example of an undecidable proposition is...

Consider the statement (C) 'A city will never be built on this spot'. Even if we have an oracle which can answer every question of the

> kind 'will there be a city here in 1990, in 2100?' etc. we might never be in a position either to declare the statement true or to declare it false.

Dummett concludes that 'either a city will never be built on this spot or a city will some day be built on this spot' is not a valid statement since neither of the two limbs is effectively decidable.

But the matter *is* effectively decidable, and by 'declaration' no less; for what Dummett forgets here are *performative utterances*, and with that the reading of 'a city will be built on this spot' as an expression of will. If some sovereing (a king, a parliament, an electorate) decrees that such a thing will be so, then it will be so: such a person is in a position to determine the course of events, not by having *evidence* for the course, but by *choosing* the course. He is then determining the exact reference of 'this spot' (or its ϵ-term equivalent) see Sommers, *op. cit.*, p. 315.

This point is also important in connection with the Logic of Time—this is still two-valued, even though we often have the power to bring things about. Dummett's other case mentioned above is important in connection with Subjunctive Conditionals, which I will look at in more detail in section 2. Sommers had said with regard to this other case (*op. cit.*, p. 311):

> The works of fiction provide one rich field for the application of the distinction between being potentially P and potentially not-P, but being determinately neither. But it is not the only one. In an interesting paper, Michael Dummett has suggested that even singular propositions about the actual past may have an underdetermined subject. Dummett imagines a man Jones, now dead, whose bravery was never put to any test during his lifetime and he considers the sentence 'Either Jones was brave or he was not brave'. Dummett sets up two positions, one maintained by A, the other by B, concerning the validity of his sentence. He assumes that none of the facts available to us would enable us to project how Jones would have behaved in a test situation. According to A, the disjunction is then not valid since neither

of its limbs is 'decidable'. According to B, the disjunction is valid since one of the two propositions must be true even though 'its truth may lie in a region accessible only to God which human beings can never survey'. Dummett does not clearly consider the possibility that Jones may actually be underdetermined to either bravery or to its opposite, but this possibility is certainly coherent.

Indeed it is undeci*ded* whether Jones was brave or was not, but our inability to *project* (i.e. deterministically predict, or prove) how Jones would have behaved in a test situation still leaves the disjunction decidable, and hence 'valid', since the sovereign body in question, namely Jones himself, had the power, by pure and simple will, to decide it. He would then be settling (without evidence) the exact reference of 'Jones', i.e. determining which of '$j = \epsilon x(x = j.Bx)$' and '$j = \epsilon x(x = j. - Bx)$' is true. Note that Jones' boast, or wimper, 'If my bravery is tested, it will pass/fail' does not settle the matter, since the sincerity of such an intention itself could only be tested by the actual event, and, by hypothesis, there was no such event, so the sincerity, along with the bravery, remains undecided.

This particular point about decidability, incidentally, might remind one of the well-known fact (see Kurt Gödel's "An Interpretation of the Intuitionistic Sentential Logic" in *Philosophy of Mathematics*, J. Hintikka (ed.), Oxford, 1969) that Intuitionistic Logic, in which the Law of the Excluded Middle is commonly said not to hold, and which has occupied Dummett for much of his career, is more properly understood as the logic of 'It is proved that p', than of plain 'p', itself. Certainly we might have $-Prp. - Pr - p$, but that does not stop its being the case that $Pr(p \vee -p)$, and hence it does not stop its being the case that $p \vee -p$. Why Intuitionistic Logic continues to be thought of as an alternative *propositional logic*, rather than a (mis-symbolized) *modal logic*, is a mystery, as a result of this: it certainly involves no arguments against the Law of the Excluded Middle—when properly understood. But the general point is that the Law of the Excluded Middle is saved, in a variety of circumstances, by resort to *choice*: symbolizing fictions by means of ε-terms enables classical logic to encompass them, without difficulty, and it also enables other, related, entities—like unbuilt cities, and untested men—to be brought within the same aegis. This is even the case in the third type of statement which Sommers (*op.cit.*, p. 317) classifies as a threat to the Law: where the predication is a category mistake. For it is not undecidable whether, say, Wednesday is fat or lean—the judgement that it is or is not certainly *has no basis*, but, undoubtedly, it may be, and is, still made (see, for instance, Roger Scruton's *Art and Imagination*, Methuen, 1974, p. 50f).

Likewise with the Sorites Paradox: we know, say, that F0, but that −F100, so logic tells us that (En)(Fn.−F(n+1)), and the question is how to settle what 'εn(Fn.−F(n+1))' refers to: but the only difficulty with this decision is accepting it must be *arbitrary*: given that, it couldn't be easier to settle the matter.

Hence the Sorites Paradox is resolved simply through the possibility of sovereignty and fiat, as with Dummett's 'a city will be built on this spot', and Jones' 'I am brave'. Crispin Wright recognizes this possibility, but discounts it, in his consideration of the paradox ("Language Mastery and the Sorites Paradox" in G. Evans and J. McDowell (eds.), *Truth and Meaning*, Oxford, 1976, p. 229):

> What is involved in treating these examples as genuinely paradoxical is a certain *tolerance* in the concepts which they respectively involve, a notion of a degree of change too small to make any difference, as it were. The paradoxical interpretations postulate degrees of change in point of size, maturity and colour

which are insufficient to alter the justice with which some specific predicate of size, maturity or colour is applied. This is quite palpably an incoherent feature since, granted that any case to which such a predicate applies may be linked by a series of 'sufficiently small' changes with a case where it does not, it is inconsistent with there being any cases to which the predicate does not apply. More exactly, suppose φ to be a concept related to a predicate, F, as follows: that any object which F characterizes may be changed into one which it does not simply by sufficient change in respect of φ.

Colour, for example, is such a concept for 'red', size for 'heap', degree of maturity for 'child', number of hairs for 'bald'. The F is *tolerant* with respect to φ if there is also some positive degree of change in respect of φ insufficient ever to affect the justice with which F applies to a particular case.

In essentials, then, the Sorites Paradox interprets certain vague predicates as tolerant. But this might seem a tendentious interpretation. Not that there is any doubt that the predicates in question do lack sharp boundaries; and the antiquity of the paradox bears witness to how easy it is to interpret this as involving the possession by these predicates of reapplication through marginal change. But is this a correct interpretation? Because 'heap' lacks sharp boundaries, it is plain that we are not entitled to single out any particular transition from n to n+1 grains of salt as being the decisive step in changing a heap into a non-heap; no one such step is decisive. That, however, is not to say that such a step always *preserves* application of the predicate. Would it not be better to assimilate the situation to that in which bordering states fail to agree upon a common frontier? Their failure to reach agreement does not vindicate the notion that e. g. a single pace in the direction of the other country always keeps one in the original country. For they have at least agreed that there is to be a border, that *some* such step is to be a decisive one; what they have not agreed is where. If we regard the predicates in the example in terms of this model, we shall conclude that their vagueness is purely a reflection of our intellectual lazyness. We have, as it were, decided that a disjunction is to be true—at some stage n grains will be a heap where n – 1 grains will not—without following up with a decision about *which* disjunct is true. On this view, the notion that these predicates are tolerant confuses a lack of instruction to count it the case that a proposition is false with the presence of an instruction to count it as true.

But, also, by definition, the notion that these predicates are 'tolerant' presupposes that the justice with which F might apply settles whether it *does* apply, and there need be no special justification, any more than rationality, predictability, or evidence for something being F rather than non-F.

2

Now, as we have seen, Jones' bravery (unlike, say, Descartes' ductility, and Ryle's brittleness) may be in this class. If such a moral, i.e. will-based, quality has been put to the test it will either have shown itself or not; but it might not have been put to the test, in which case it would be unverified, and (without some structural basis, as with ductility and brittleness) there could only be speculations on the matter. Some writers, however, have hoped to make such a quality determinate, even in this case—by introducing counterfactual conditionals which are *verified*. Here is M. Loux, in *The Possible and The Actual*, Cornell, 1979, p. 32, on the general analysis of counterfactuals.

> While the propositions expressed by
>
> (15) If Nixon had not resigned, there would have been a constitutional crisis
>
> and
>
> (16) If the Blue Jays were to win the pennant, Toronto would go wild
>
> are not explicitly modal, it is notorious that they are like explicitly modal propositions in resisting analysis in terms of the machinery afforded by strictly extensionalist logic. And when we reflect on the fact that claims of the form 'If it were (had been) the case that p, it would be (would have been) the case that q' are not claims about how things have actually gone, we are likely to conclude that the similarity here is no accident, that counterfactual discourse resists an extensionalist analysis for precisely the reason explicitly modal discourse does: coun-

terfactual claims are about things (possible worlds other than the actual world) that go beyond the ontology required for standard extensional discourse. In recent years, a growing number of philosophers have tried to give substance to these intuitions. Counterfactual discourse, they have argued, is indeed discourse about possible worlds; but they have insisted that while counterfactual discourse agrees with explicitly modal discourse on this score, there is an important difference. Ascriptions of modality (whether *de dicto* or *de re*) involve quantification over *all* possible worlds; but when we make some particular counterfactual claim, the reference to possible worlds is more narrowly circumscribed. When I say that if the Blue Jays were to win the pennant, Toronto would go wild, I am not saying that in every possible world where the Blue Jays win the pennant, Toronto goes wild; for there obviously are possible worlds where the Blue Jays win the pennant and few, if any, of the citizens of Toronto find the event interesting. My claim, these theorists suggest, has its eye to just one possible world, a world that is very 'close' to, very similar, to the actual world. I am talking about that possible world which is as like the actual world as is compatible with the Blue Jays winning the pennant in it, and I am saying that in that world, Toronto goes wild.

Now there is a detail, in this analysis, which may be immediately questioned, for it holds that 'If p were true then q would be true' is '$V((p.q), w_j) = 1$' where 'j' satisfies some relation, say, Sio, in which 'o' indexes the actual world. If the former aspects of this analysis were correct we could not make counterfactual conditions with *impossible* antecedents, since '$V((p.q), w_i) = 1$' entails '$M(p.q)$'. We must therefore write '$V((p \supset q), w_i) = 1$' as the formal expression for the counterfactual; but, more importantly, we must add to it no governing condition like 'Sio'. This is because any closeness is a measure of the probability that the world in question is actual, and hence is a *qualification* of the conditional, not a constituent in it. For, while we can say 'It is probable that if p were the case, q would be the case', i.e.

$$\text{Prob } [V((p \supset q), w_i) = 1 / w_i \epsilon W] > 1/2$$

the conditional 'if p were the case, q would be the case' is thereby just the above contained part—with 'j' indexing some (context-dependent) supposed case. It is well known that this style of analysis is what holds with the limiting *necessitation* conditionals, i.e. ones for which the probability is 1, since then the appropriate probability statement is equivalent to

(i) $(V((p \supset q), w_i) \mathrel{\neg\!\!=} 1)$,

and this *proves* that the non-necessitated, particular counterfactual is what is there quantified.

But, if so, then, in the absence of general laws, counterfactual conditionals are, by their nature, as Quine said, unverified, and each speaker can make up his own story, about his own chosen possible world, at will. Whether 'If Nixon had not resigned, there would have been a constitutional crisis', 'If the Blue Jays were to win the pennant, Toronto would go wild', 'If Jones' bravery were put to the test, it would pass/fail' are asserted, or denied, is therefore as one fancies (supposes!): the only way such imaginings could have been realized was through the intercession of the appropiate sovereing powers—the constitution defenders, with Nixon, the Toronto populace with the Blue Jays, and Jones with regard to his bravery—and the opportunity for that has passed, and cannot be post-judged anymore than it could be preempted. But if we cannot adjudicate between

If Bizet and Verdi had been compatriots, Verdi would have been French

and

If Bizet and Verdi had been compatriots, Verdi would not have been French then why should we say either, i.e. why should 'Conditional Excluded Middle' hold?

Stalnaker, from whom Loux's analysis of counterfactuals derives, says ("A Defence of Conditional Excluded Middle" in *Ifs*, W. L. Harper, R. Stalnaker and G. Pearce (eds.), Reidel, 1981, p. 102):

> If President Kennedy had not been assassinated in 1963, would the United States have avoided the Vietnam debacle? It is a controversial question. We will probably never know for sure. If we could look back into the minds of President Kennedy and his advisors, if we could learn all there is to learn about their policy plans and priorities, their expectations and perceptions, then maybe we could settle the question. But on the other hand, it could be that the answer turns on possible actions and events which are not determined by facts about the actual situation. In that case we *could* never know, no matter how much we learned. In that case, even an omniscient God wouldn't know. If this is true, then our failure to answer the question is not really an epistemic limitation, but we still use the language of knowledge and ignorance to characterise it. Even when we recognize that such a question really has no answer, we continue to talk and think as if there were an answer that we cannot know. This is, I think, because we tend to think of the counterfactual situations determined by suppositions as being as complete and determinate as our own actual world.

Indeed, the powers in Stalnaker's question, namely Kennedy and his advisors, may have had nothing in mind, and, even if they did have some intention or plan, its sincerity or applicability could only have been tested by the actual event, so what supports 'the language of knowledge and ignorance' is not any 'closeness' condition (*op. cit.*, p. 89), which, in any case, is technically difficult to maintain (*op. cit.*, pp. 97–98), but merely the fact that free agents have the general ability to settle such questions as and when they arise. Indeed consideration of the (totally) free agent case alone rules out 'closeness' as relevant, since with a quite arbitrary sovereign power the Principle of Indifference holds and no one possible world is any 'closer' than the next.

Presuming Bizet and Verdi were cosmopolitan enough to be indifferent to their nationality, their case is in this optional class; though, given they were in fact late nineteenth century composers, this hypothesis is unlikely, and some measure of the respective strengths of their patriotic allegiances would be necessary to rank the probabilities of the above two conditionals. But, even then, as before, neither conditional could be ruled out—although one or the other must be ruled in, since, in every possible world, '$p \supset q$' or '$p \supset -q$' is true. What one supposes is therefore a bet which is never verified—and some punters like to back favorites, others outsiders.

There is no nationality which Verdi and Bizet, in fact, share, i.e. $-(EN)(Nv.Nb)$; so that means 'the nationality they share' i.e. '$\epsilon N(Nv.Nb)$' could (logically) be any such. In another Stalnaker example (*op. cit.*, p. 97) a line is less than 1 inch long—so it has no length greater than that, i.e. $-(E\epsilon)(La\epsilon.\epsilon > 1)$; but that means 'its length greater that one inch' i.e. '$\epsilon\epsilon(La\epsilon.\epsilon > 1)$' could be any length. Clearly it is only within a calculus which allows descriptions not to describe that we can start to understand these things.

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RESUMEN

En este artículo muestro cómo varias amenazas a la ley clásica del Tercero Excluido pueden atacarse mediante el cálculo-ϵ de Hilbert y la noción de elección que comprende. Tras introducir informalmente este cálculo, expongo su utilidad en la teoría de la referencia ya que necesita tener términos que denoten, mismos que no necesariamente describen lo que denotan. Así, es posible, dentro del cálculo-ϵ de Hilbert, que '$F\epsilon xFx$' sea falso (puesto que es exactamente equivalente a '$(Ex)Fx$') y, por consiguiente, 'ϵxFx' denote un objeto al que 'F' puede no describir. Ésta es una teoría no-clásica de las descripciones, pero nos proporciona inmediatamente una teoría de las ficciones dentro de la lógica clásica: ya que, al tomar 'ϵxFx' como 'la F', podemos decir invariablemente $G\epsilon xFx \vee -G\epsilon xFx$, exista o no $F\epsilon xFx$, es decir, haya o no una F. Es así como la ley del Tercero Excluido se salva aquí, y un argumento similar salva a la ley en el caso de contingencias futuras y circunstancias pasadas contrafácticas. Además, el principio según el cual $G\epsilon xFx$ es verdadero o falso *por elección* cuando $-F\epsilon xFx$ indica defensas 'electivas' similares de la ley con respecto a errores categoriales y de vaguedad. El artículo finaliza con un análisis más detallado del caso contrafáctico abordando directamente la cuestión del 'Tercero Excluido Condicional'.