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NATURAL NUMBERS

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The horns of Benacerraf's dilemma are sharp.[1] On the one hand, pure mathematics blends into applied, which merges with physics, which modulates into chemistry, and so on perhaps through economics, history and literary criticism, in such a way that a single univocal notion of truth emerges right across the whole scheme of human knowledge. As truth in history requires reference to kings and countries, so truth in mathematics requires reference to numbers and functions. But while kings are made of flesh and bone, the numbers required among the values of the quantified variables of the theorems of pure mathematics for their truth seem neither physical nor mental but abstract. On the other hand, we are happiest in granting that a person knows a truth when we understand, at least roughly, how that person so engaged with the subject matter of the truth that he was thereby justified in believing the truth. Perception is the only generally accepted mode of such engagement with independent subject matter, and Grice argued persuasively that perception is by nature causal.[2] But it is as close to axiomatic

[1] Paul Benacerraf, "Mathematical Truth", *Journal of Philosophy*, 70 (8 November 1973), 661–679.

[2] H.P. Grice, "The Causal Theory of Perception", reprinted in *Perceiving, Sensing and Knowing*, ed. Robert J. Swartz, Doubleday, New York, 1965, pp. 438–472.

as metaphysical claims get that objects as abstract as numbers would be utterly inert. So, and this is the dilemma, what seems necessary for mathematical truth seems also to make mathematical knowledge impossible.

Dummett distinguishes among atomic, molecular and holistic accounts of meaning.[3] We might make parallel distinctions among stories about truth. On Wittgenstein's picture in the *Tractatus*, any two elementary propositions are independent, and each is true all by itself if and only if the bearers of the names in it are configured as are the names; that we might call an atomic conception of truth. On a Tarsky-style story, sentences are well-ordered by something like grammatical complexity and the inductive conditions for the truth or falsity of one sentence may depend on others, but only on others lower in the ordering; that we might call a molecular vision of truth. On an absolute idealist conception of truth like Hegel's, Green's or Bradley's, truth is a matter of incorporability into a coherent monolith of lore; that we might call an holistic story of truth. We might also make parallel distinctions among accounts of justification. One sort of empiricist might count a person as knowing a truth only if he has experiences constituting, according to verificationism, the truth-conditions of that truth; such a truth-by-truth picture of justification might be called atomic. On a foundations story, truths are well-ordered by something like epistemic priority, and while some truths may be justified by inference from others, they may only be so justified from truths lower in the ordering; this we might call a molecular account of justification. Quine once wrote that no statement is any more intrinsically a postulate than is a place in Ohio a

[3] See Michael Dummett, "What is a Theory of Meaning?", *Mind and Language*, ed. Samuel Guttenplan, Oxford, 1975; and "What is a Theory of Meaning? (II)", *Truth and Meaning*, eds. Gareth Evans and John McDowell, Oxford, 1976.

starting point;[4] on such a conception truths are justified only in large and heterogeneous bodies that blend into the whole web of belief, and such a story of justification we may call holistic.

Has there been a silent preconception that truth and justification must share a structure? It would certainly seem that if an atomic account must hold for both, then Benacerraf's dilemma is insoluble, for it is all but axiomatic that we do not inter-act causally with, and so do not experience, the objects required for the truth of the theorems of pure mathematics. If a molecular account must hold for both, then the tighter the inferences required for justification the less likely Benacerraf's dilemma is soluble, since what we can tell just by perception of objects alone seems hardly to entail the theorems of pure mathematics. So at one point Quine suggested joining a Tarsky-style molecular story of truth with a holistic account of justification.[5] The basic epistemic principle here is that we are justified in believing what we need to suppose in order best to explain what we observe. Harman called this principle inference to the best explanation,[6] and it may be what Peirce meant by abduction. So since, according to Quine, we need some mathematics to do natural science of any sophistication, we have the same sort of reason for believing in numbers as we have for believing in the dinosaurs we suppose to explain fossils. The holism of this pattern does not require that we ever do, or even could, see the dinosaurs and numbers about which paleontology and mathematics record truths.

There are some obvious questions about Quine's combination of molecular truth and holistic justification. What about the

[4] See W.V. Quine, "Two Dogmas of Empiricism", *From a Logical Point of View*, 2nd ed., Harvard, 1961; also, W.D. Hart, "Access and Inference", *Aristotelian Society*, Supplementary Volume, LIII (1979), 153–165.

[5] *Ibid.*

[6] Gilbert Harman, "Inference to the Best Explanation", *The Philosophical Review*, 1965, 88–95.

justification of so much mathematics notoriously unsullied by application? What about the rôle of proof in the phenomenology of mathematical justification but missing from Quine's story?[7] What about Hartry Field's attempt to deny Quine's thesis of the indispensability of mathematics for science by trading numbers in for less abstract geometrical objects?[8] Let us put these questions aside. There is also the deeper question of the significance for knowledge and truth of attributing to them different structures. For example, if they are laid out differently, might they so diverge as to threaten the doctrine that someone knows that *p* only if it is true that *p*? In what larger patterns might truth of one structure and knowledge of another be re-united? Let us bracket these questions, at least for the moment.

Let us instead review the second horn of Benacerraf's dilemma. This was epistemological, or at least presented as such. We are happiest, we said, with granting a person knowledge of a truth when we understand, at least in outline, how that person so engaged with the subject matter of the truth as thereby to be justified in believing the truth. Perception, we said, is the only generally accepted mode of such engagement, and we agreed with Grice that perception is by nature causal. But, we granted, it is all but axiomatic that objects as abstract as numbers would be utterly inert. The steps here from engagement and justification to the causality of perception are plainly epistemological (at least in part). But the utter inertness of utter abstracta like numbers seems wholly metaphysical. For example, inference to the best explanation has yet to receive its naturalization papers in epistemology. How might abduction, or its ancestor in Leibniz, the principle of sufficient reason, be fitted into a natural history of mathematical knowledge? If causation does not bind people and numbers together, can inference made by people

[7] These two objections are answered in Hart, "Access and Inference", cited in note 4 above, pp. 157–158.

[8] Hartry Field, *Science without Numbers*, Blackwell, 1980.

from their best explanation to justified belief in abstracta like numbers be naturalized?[9]

What is really going on here might be called the return of cosmology. Once upon a time, metaphysics had two departments, ontology and cosmology. Ontology asked after the most basic kinds of things that there are. A liberal and fictional Plato might have answered that matter, mind and abstracta like forms or numbers are the basics, their fundamentality lying in the independence of each from the existence of examples of the other two. (The real Plato seems less liberal toward matter.) A materialist was someone who counted only matter as basic. By contrast, cosmology asked after the form or pattern into which the basics go to make the world. Here it seems shrewd to recall Kant's discussion at the very end of his first *Critique* and throughout his third of what he called the systematicity of nature. To be sure, we have lost Kant's confidence that we know a priori that, or that it is a necessary truth that, there is a single unified system into which everything there is falls. Nevertheless, the principle of the systematicity of nature seems to retain a regulative and critical force in our judgement. For an ontology that asserts basic categories, between which no systematic connections are possible is for that reason a suspect ontology. Even if the physicists have highjacked the word "cosmology", its old denotation still sits in judgement among our ontological categories.

Cosmology was a lively field in nineteenth century philosophy. Naturalists, who where often materialists, took the field under a banner we might picture as inscribed with a slogan from

[9] In *The Roots of Reference* (Open Court, La Salle, Illinois, 1973), Quine at least seems to assimilate the naturalization of the epistemology of abstracta to use/mention confusions. But to reason by confusing use and mention is fallacious, so such naturalization might seem to leave abstracta illegal immigrants.

Hume, "Causation is the cement of the universe."[10] This is not, however, to say they all followed Hume in accounting for causation exclusively in terms of temporal priority, spatial contiguity and constant conjuntion. Helmholtz, for example, probably anticipated Quine in thinking of causation much more naturally than Hume as the flow of a strictly conserved quantity, which we now call energy.[11] A critic, like T.H. Green, of the naturalists did not quite so much deny Hume's slogan as ask after its presuppositions and their naturalness. An account of causation like Quine's or Helmholtz's is based in conservation principles. These take many, and subtly related, forms, but the root ancient idea is that *ex nihil nihil fit*, you do not get something from nothing, there is no magic (in which rabbits really appear in empty hats or in which pretty girls really just vanish). Probably we do not know a priori that, nor is it a necessary truth that, *ex nihil nihil fit*; but probably we know that, and it is a truth that that, from nothing nothing comes. The sad wisdom, Green might have said, that there is no real magic provides the form of our understanding of causation, and through that form nature in the last century revealed the conserved flow of energy. That sad wisdom also articulates a presupposition of the question, "Where and how and why did such-and-such come?" namely, that it came at all, and so sets a standard for the enterprise of explanation. In *Pursuit of Truth* Quine grants that successful prediction is a, and perhaps the, test of scientific theory, but he rightly points out that one of its major purposes is understanding.[12] We might add that describing natural history in conformity with conservation is, in our best judgement at present, central to understanding nature; conformity with con-

[10] David Hume, abstract of *A Treatise of Human Nature*, ed. L.A. Selby-Bigge, 2nd ed. P.H. Nidditch, Oxford, 1978, p. 662.

[11] See Quine, *Roots of Reference*, pp. 4–8, and W.D. Hart, *The Engines of the Soul*, Cambridge, 1988, chap. 5.

[12] W.V. Quine, *The Pursuit of Truth*, Harvard, 1990, pp. 2 and 20.

servation is part of the form we now expect or require of systems of explanation. Prediction is another matter. Old Faithful, the famous geyser in Yellowstone Park, erupts once every sixty-six minutes. That regularity plus the knowledge that it has just erupted is enough all by itself to predict eruptions of Old Faithful for quite a while. But it does not seem to be enough all by itself to explain or understand those eruptions. (Where do the heat and the water come from? Why does the heat build up for a while, and then the water burst out suddenly?) What is missing for explanation and understanding seems to be theory, a systematic (quantitative) account of what a geyser is and how it works. Such a theory might be tested by how well it predicts eruptions of Old Faithful, but evidence is not content, while explanation and understanding seem to be matters of content. Plate techtonics includes a theory of what earthquakes are and how they work. It would be at least prohibitively expensive to fill in the detail that might, for all one knows, enable us in 'principle' to predict earthquakes, but that lacuna does not seem to shake our confidence in the explanation and understanding of earthquakes by plate techtonics, for which other evidence, like the fit between South America and Africa, is available. In sum, in the absence of theory, prediction does not suffice for explanation and theory may suffice for explanation of a phenomenon without prediction of that phenomenon. Perhaps we should also deny that there is a unit of explanation, that nomological deductions are to explanations as sentences are to truths; instead, explanation is developed in systems whose growth and revision deepens and extends understanding of the world.

We have now associated explanation more intimately with understanding than with prediction. Distaste for this greater intimacy may betray a horror of subjectivity, as if the value of objective science might boil down to the thrill of an aha! —*Erlebnis* and the buzz expressed by "Eureka!" Hegel may have developed the association between explanation and understanding under the slogan that the real is the rational by

taking the rational as what makes sense and construing the systematic unity of the world in terms of the unity of a mind, *Geist*, in which such sense is made, but we seem to have lost Hegel's confidence in God or historical human culture. It would take courage, or an over-developed professional death wish, to defend absolute idealism these days (though Hegel might explain the granting of such a wish by appeal to *Geist*). So it might be shrewd to exclude subjectivity as much as possible from the intimacy between explanation and understanding. Tastes are subjective, even rather general ones like the affection for chocolate, for if one person prefers vanilla to chocolate while another has the reverse preference, their difference does not show that at least one of them is wrong, but is rather to be savoured. But understanding is a variety of knowledge, and while knowledge includes mental states like belief, it also requires truth as objective as one likes. There are no pairs of people one of whom knows that the moon is cubical but the other, that it is not; of pairs claiming such knowledge at least one is wrong. So knowledge, and thus understanding, can be as objective as truth, that is, as answerable to objects whose existence and nature is independent of how belief about them would have them. What is explained can be understood without necessarily becoming for that reason subjective or mental.

This observation amounts to no more than distinguishing between our systematic understanding of what there is and the system or systems into which, according to that understanding, what there is falls. That distinction is hardly hot news. The tough minded among us often seem to think of our best understanding of what there is as hard science. Deans and bursars drawing up budgets conventionnally divide hard science into natural science, like physics and chemistry and geology, and mathematics, usually including nowadays some provision for mathematical logic as well. Is this division also of cosmological significance? The cosmological slogan culled from Hume had it that causation is the cement of the universe. It is pre-

sumably the subject matter of natural science, the quarks of physics and the molecules of chemistry and the continents of geology and the galaxies of astronomy, that causation cements. If we also take seriously the numbers, sets and functions of mathematics, then it seems a cosmological axiom that it is not causation that binds them up, not least because they lack location in the spatio-temporal arena of causation. In our time set theory has become a general framework for the exposition of this or that part of mathematics, and there is a certain systematic elegance and economy to be had by finding set theoretic surrogates for the numbers, functions and spaces of the various special branches of mathematics. So it might bring a certain order to one's conception of mathematics to ape Hume by saying that membership is the cement of the more abstract reaches of what there is. But of course, if we do say this, we are obliged in good intellectual conscience to recognize that we have restricted his slogan, for we may now say no more than that causation is the cement of at most the more concrete reaches of what there is.

The resulting cosmology recognizes two basic sorts of things, concreta stitched up by causation, and abstracta knit up by membership. This cosmology might remind one of middle Plato's material particulars and abstract forms, though it is more tolerant of concrete matter than Plato often seems. He wrestled heroically trying to align, and bridge the gap between, the two sorts and his attribution to Parmenides of the third man argument might be taken to record his recognition of the collapse of the resemblance bridge between concrete particulars and forms, for it seems to be the resemblance story that calls for the self-predication doctrine according to which the form of a man is a man and which then issues in the third man argument. But Whitehead said that all philosophy is a footnote to Plato, and filial piety or honest editing may be served by thinking of membership as anticipated by Plato's participation. Note here that by our present lights, membership links not only sets to

sets but also non-sets to sets. Each thing that is not a set is an urelement and a member of many sets. So this cosmology need not admit two disconnected and unrelated fragments of what there is, but may appeal to the thread stitching together its more abstract denizens to sew a seam between them and its more concrete denizens. (Of course it may also retain more specialized stitches, like the numbering relation between nine and the members of the set of planets, or the evaluation relation between an abstract function and a concrete argument, for those reluctant to reduce mathematics to set theory.) One hopes that Plato might have taken pleasure in the thought that while causation binds matter and membership binds sets, it is also membership that binds matter to sets and so paves a way for the unity of the universe.

There may be another pleasure for Plato lurking here too. Mostly we shall omit modality here, but let us digress for a bit and glance briefly at possibility and necessity. The obvious simple idea of dependency between objects is that an object, A, depends for its existence on the existence of an object, B, if and only if it is possible that B exist but A not while it is necessary that if A exists, then B does too. It is an ancient and honourable dogma that if there are very abstract objects like numbers, pure sets and functions, then they exist necessarily; while each concrete object like a galaxy or glacier that actually exists exists only contingently and could have failed to exist. It follows from the simple idea of dependency and the ancient dogma that all concrete objects there are depend for their existence on all the abstract objects there are. This cosmological priority is distinctively platonic, and is a feature of platonism that particularly upsets philosophers like Aristotle who begin with a tacit promise not to frighten the horses. Now let us holster our modalities.

So far our cosmology has concentrated on concrete matter and abstracta like numbers and sets. This concentration neglects an important footnote to Plato by Descartes. It seems

pretty clear that in the *Phaedo*, Plato has Socrates make dualist claims about the mind and the body, that is, that neither depends for its existence on that of the other, but that each could exist although the other not. It is often perplexing how seriously we are meant to take the arguments Plato puts in Socrates' mouth. But Descartes has a very good argument for dualism: what one can imagine is possible; and one can imagine being disembodied, and thus could exist although one's body not, which is the crucial thesis of dualism. This is hardly the occasion on which to defend this cartesian footnote to Plato in detail,[13] but a few remarks on its cosmological aspects may be in order. It certainly seems obvious that there are causal connections between mind and matter: if you punch me in the nose, which is physical, it will hurt, which is mental; and if I decide to snap my fingers, which is mental, my fingers will snap, which is physical. From very early on, understanding how there could be such causal connections between things as different as a dualist thinks mind and matter are has been seen as a, if not the, crucial problem for dualism. Observe that this is a cosmological problem. A way into this problem is secured by noting that it is as much a problem about the nature of causation as it is a problem about the natures of mind and matter. If, for example, one thinks that causation pretty much boils down to brute constant conjunctions between events of certain types (whether constancy is larded over with the subjunctive mood or not), then since there seems no more problem about constant conjunctions between mental and physical events than about such conjuntions between physical and physical events or between mental and mental, the problem evaporates. So whether there even is a problem of mind-body interaction is sensitive to articulating an understanding of causation outside the main stream of philosophical thinking about causation since Hume. Although its roots seem to go very deep into the history of our

[13] For more, see Hart, *The Engines of the Soul* cited in note 11 above.

intellectual culture, the nineteenth century natural scientists' idea of causation as the flow of strictly conserved energy, which is an articulation of the medieval idea that there is as much reality in the cause as in the effect and of the even older idea that you do not get something for nothing, is an understanding of causation foreign to Hume, since the conserved quantity energy need not to be directly observable, that makes the problem of mind-body interaction properly difficult: for what could flow between things as different as the dualist takes minds and bodies to be without impossibly partaking in the natures of both? Yet the energy flow view of causation not only makes the interaction problem properly difficult; it also describes in outline what would be required to solve that problem, and so points ways towards trying to solve it. Maybe the dualist can solve the interaction problem. We will not try even to sketch such a solution here. But note that should some such solution work, the cement of universe may help to unify minds separately and to bond them into the causal nexus of nature, which would be a triumph for our conventional cosmology.

But need causation be the only mortar of the mind? Thought has its normative dimensions, and it is a part of the business of reason to superintend the rationality of judgement and inference. If we were always sound and complete thinkers, it might be plausible to suppose that there are causal laws of thought mirroring the laws of logic and whatever other legitimate patterns of reasoning there may be. But each of us learns that he is neither infallibly sound nor infallibly complete at thinking things through correctly; at the chalk-face of thinking we do not always live up to the ideals of rationality. Our recognition of this falls under the critical, normative function of reason, and so betrays our recognition of at least some of the ideals of reason. How good are the prospects of naturalizing reason's recognition of the ideals of reason?

Since Aristotle, logic, nowadays cast as first order quantification theory with identity, has seemed our surest canon of the

ideals of reason. Since Plato, the epistemology of logic has been assimilated to the epistemology of mathematics. Logic and mathematics are the two systematically developed and agreed bodies of knowledge most likely to attract the adverb "a priori" and the only two. Suppose, with Plato, that mathematics, such as number theory, answers for its objective truth to how it is among independent objects, the natural numbers. Philosophers as different as Plato and Kant have inferred that there must be some direct epistemic access from the mind to the subject matter of mathematics. Plato and Gödel model such access on perception, for perception is the only process linking belief to independent subject matter whose exercise is agreed to justify belief. But this seems hopeless, for it is in the nature of abstract objects to be inert, and so immune to causal interaction, which includes perception. Kant, on the other hand, takes the exercise of understanding and the forms of intuition to be at least partly constitutive of truth known a priori. But transcendental idealism, like all idealism, seems too much to undermine the objectivity and independence of the truth for whose knowledge it offers such certainty as there is in self-knowledge. (Analyticity and truth by convention were just linguified or socialized idealisms of logical and mathematical truth.) It is high time that grown-ups put away, perhaps sadly, the illusion of a direct, natural epistemic grasp by the mind on numbers. The world just isn't that unified.

It is here that a naturalist looks to be able to take a kind of revenge on Plato. For the mind has causal access to matter by perception, and in order to make sense of what it thus perceives needs to suppose things like dinosaurs, space, fields and numbers that it does not perceive, but nonetheless thereby acquires reason to accept. In making sense of a world in which it is causally embedded, the mind acquires an indirect and inferential epistemic access to causally inert objects to which it has no direct epistemic access. Matter is an epistemic bridge from the mind to numbers. There is a toll, namely, the absolute certainty

of mathematical knowledge to which we supposed ourselves entitled, but we had never paid in justification for that absolute certainty anyhow. Besides, the confidence we once placed in the comprehension principle of naïve set theory, the parallels postulate of geometry, and Euclid's principle that the whole is always greater than any of its proper parts shows that we con ourselves about certainty. An intuition is just a belief for which we ought to ahve justication but don't; ub-thumping about certainty and the a priori just fills the gap with noise.

But logic is ubiquitous; *modus ponens* is as good in physics as in number theory. The doubt here may concern the generality of logic, or its normative power. Let us consider generality first. Wherever we can check independently on the premises and conclusion of an instance of *modus ponens*, we have learnt that it preserves truth, whether the subject matter is quarks or quotients. We have come to characterize this uniformity by semantic ascent, by syntactical relations among conditional sentences and their antecedents and consequents, by the semantic conception of truth, and by the law that when a conditional and its antecedent are true, so is its consequent. To suppose that *modus ponens* reveals a structure in the world over and above either the truth functions or the structure revealed by the premises and conclusion, if true, of an instance of the *modus ponens*, seems to require an apparatus of something like facts or propositions. Maybe there are such things, but for us to be justified in believing in them, they should be needed to explain more outside the sphere or spheres whence their introduction was motivated than have the objects of the propositional attitudes, the meanings of sentences, or the bearers of truth values implicated in the liar paradox. If making sense of the mind required treating propositional attitudes as relations between people and propositions, then there might be a serious question whether those propositions are also the propositions in another of their putative rôles like meanings of sentences composed of meanings of constituents of those sentences, or as

distinguished from sentences as, according to ZF, solving the paradoxes of set theory requires distinguishing sets from predicates. A crux here would be whether there are any interesting (explanatory), unexpected (not built in) and testable interactions among the structures in which propositions are thus arrayed. But we have yet to make much systematic testable sense even of just ourselves, and if propositions are as abstract as usually supposed, it is hard to see how the epistemic dimensions of propositional attitudes apparently required by consciousness could be naturalized. While sets have been pressed into service outside set theory, thus providing tasks and objectives by which to design and judge such theory, proposition theory seems more like an engine idling; and if propositions were like sets subject to limitation of size, what would become of the generality of logic?

Turning now from generality to power, it might be shrewd public relations at least to say that logic has the normative virtue, but then go on to explore the significance of this lip service to convention. Once upon a time, we used to distinguish between regularities and rules. The geyser Old Faithful erupts once every sixty-six minutes or so. That is a regularity, for if a person says so and then Old Faithful goes off every five minutes or sits quietly for days at a time, it is not Old Faithful who is at fault but the speaker who is in error and whose words are false. The English verb "try" takes the infinitive, so one may say one will try to fly by flapping one's arms, not that one will try *and* so fly, as if the effort guarantees success by conjunction elimination. The English adjective "different" takes the preposition "from," not "to," and certainly not the conjunction "than" as if it were the comparative of some positive adjective. These are rules of English grammar or composition, for a person who violates them does not thereby refute them, but rather shows himself to be an inadequate master of English. It is the exception that distinguishes the rule from the regularity, for an exception refutes a regularity while it is the exception

that it is at fault for flouting a rule; to make it vivid, one might say that the vector of criticism goes from the exception to the regularity, but from the rule to the exception. Sometimes norms and values find adequate expression in rules, and logic has its rules of inference.

But perhaps we can think of rules for human behaviour as regularities embedded in further regularities of critical behaviour. Let us take criticism and its recognition for granted; marks and grades may come first to the minds of teachers and students, but critical behaviour is a broad collective ranging from assault and embrace through dismissal, passing over and promotion to the subtleties of disdain and respect. It is a rule of English composition that the adjective "different" takes the preposition "from" rather than "to". It is a regularity of English speech that by and large educated and reflective English speakers say "different from" rather than "different to". It is a regularity of critical behaviour among English speakers that parents, teachers and good friends will correct those who say "different to" and will accept those who say "different from". It is a further regularity that the parents, teachers and friend of parents, teachers or friends who neglect to correct such infelicities or who attempt to correct such felicities will object to that omission or commission. And so on, probably not forever, but perhaps indefinitely far though with fading force, and maybe far and firm enough to make it a rule for good English that "different" takes not "to" but "from".

Similarly, perhaps, logic teachers and their teachers reward applications of *modus ponens*, condemn objections to applications of it, and so on, thereby making it a rule of logical inference. Of course biology teachers and their teachers also enforce similarly the doctrine that cows have four stomachs. In both cases there lies at the centre of the nest of critical behaviour a fact: cows do have four stomachs; and the consequent of a true conditional with a true antecedent is true. The embedding may be deeper and firmer in the case of *modus ponens* than

bovine ruminancy, but anyone who ever got the wrong answer in an experiment knows about the embedding of laws of nature in regularities of critical behaviour. The division of intellectual labour and the inheritance of acquired characterizations makes such enforcement sensible, at least to a certain extent, but there is no magic that renders a rule infallible, a priori or necessary just because it is a rule. Even if *modus ponens* be necessarily sound and known to be so *a priori*, neither would be explained by its enforcement as a rule.

The extent to which a regularity becomes embedded in regularities of critical behaviour, and the firmness with which such a rule comes to be enforced, seems in many cases to vary with the extent to which it impinges on systematic expositions of our lore. *Modus ponens* comes into play more often than Newton's three Laws of Motion, they are germane more often than the anatomy of cattle, and that is fitted in more often than the relatively brute sixty-six minute period of Old Faithful; indeed, one might measure the bruteness of a fact by its degree of isolation from system, and expect it to be less enforced as a rule. Rules are not a special category transcending fact. Whether, and how much, a regularity is accorded enforcement as a rule is, as Quine, Goodman and White insisted decades ago, a matter of degree, but no degree of enforcement can render a claim to truth utterly inmune from scrutiny, reconsideration, and the risk of revision. The normative power of a rule lies in the depth and strenght of the regularities of critical behaviour in which it is embedded. So we can incorporate rules into our cosmology without, at least for that reason, admitting the occult power of a priori knowledge or truth in all possible worlds, each causally isolated from the rest.

But enforcement is neither articulation or explanation. Logic was enforced for centuries before it reached the articulation of it begun by Frege and Russell. With articulation comes enlightment, the kind of understanding that Wittgenstein said consists in seeing connections, the depth of system. As we have set it

out, the epistemological engine of our cosmology seems at crucial points to have been inference to the best explanation. But that seems to be a rule more enforced than articulated. Wouldn't it be lovely to work out a statement of that rule whose terms were systematically connected with terms elsewhere in our system of the world? Can we explain the powers of explanation? Can the understanding grasp its grasp on the system of nature?

The word "system" has been, at least by and large, a term of abuse or contempt in analytic philosophy. It may be that Quine is sometimes exempted from this general analytic suspicion of system, perhaps because he articulates a hard-headed scientism congenial to our century. But mostly analytic philosophers seem to practice in a mere local and piece-meal way. Austin had a vision of philosophy as a cooperative enterprise. Individuals would work out bits here and there, the later presumably drawing on, rather than refuting, the work of the earlier, like micro-biologists mapping the human genotype. Where acknowledged experimental methods for settling questions persist, such cumulative cooperation may be feasible and worthwhile. But where micro-biology has evolved and acknowledged persistent experimental methods, analytic philosophy seems to have offered instead only what has been variously called common sense, ordinary language or intuition. Whether, and if so why, these might merit confidence seems an apt, if unanswered, question, but it seems clear that if one raises public opinion to the supreme court, then one runs a risk that oppressive fashionable orthodoxies will succeed each other as pretenders to the common law. Maybe academics can't buck the market, but is philosophy as a sort of intellectual top of the pops really what we wanted to do?

The later Wittgenstein *may* have toyed with a more radical and principled resistance to system. Yes, perhaps he was willing to grant, there are numbers, and their laws and relations are those worked out in number theory and analysis. Sure, there are objects, what philosophers call physical objects, and their laws

and relations are those worked out in physics, chemistry and so on. Sure, there are minds, just as we always thought, and maybe we'll work out a psychology of them one day. But there is no more inclusive category than these. There is no utterly general concept of an object or a thing, there is no question whether a number or a mind is an object, and there is no system of numbers, minds, stones and everything else at once to be discovered. The craving for generality is an illusion, an illusion that led to the notorious delusions of systematic philosophy in the grand matter.

It is always hard to figure out what the later Wittgenstein was on about, but these views are unattractive. A punch in the nose hurts, and some decisions issue in actions. Euclid knew that there are infinitely many primes, and there are nine major planets in the solar system. There are more sets that there are elementary particles or minds. But there are more than examples of connections between the categories that seem to demand system. For while, of course, even philosophers use the work of their predecessor, this use is selective and critical. That it should be so is inevitable certainly because even the smartest of us is fallible, and perhaps because of the underdetermination of theory by data. If even our richest ancestors leave us only a choice among bequests, some of which may be counterfeit, then the heirs must chose what to use, and it is wise for them to do so critically. If the alternatives fit the data equally well, at least as far as we can tell, then it seems inevitable that the heirs fall back on systematic considerations in making their choice. But there seems to be no convincing way to insulate sections of system from criticism generated from other sections. The physics of matter draws on the mathematics of number and space, and famously the parallels postulate of geometry was revised to suit the physics of gravity. If there is ever a scientific psychology, it seems sensible now to bet that it will draw on the sciences of matter and number, and such a merger would open now unforseen paths for mutual adjustment and revision. And, as Frege

and Russell saw it, in logic we have our best judgment of the laws of objects generally, whether mental, physical or abstract.

We began with Benacerraf's dilemma. His problem, put briefly, is that the abstract objects, like numbers, apparently required for objective truth in mathematics uniform with truth elsewhere are at least *prima facie* incompatible with the justification ultimately by perception of, and thus through causal interaction with, the subject matter of knowledge apparently required by empiricism; what makes mathematical truth possible seems to make mathematical knowledge impossible. It is pretty easy to see that there are analogues of this dilemma for the mind, for modality, for morals, and for space and time. Of course, that these dilemmas are analogous at least from a sufficiently abstract point of view is not a good reason for expecting them to share analogous solutions too. But each is a cosmological problem, a problem about how if at all separately attractive characters could be bound up in single systematic narrative of a single world, and no rhetoric about dissolving pseudo-problems seems to make them go away. Perhaps the impulse toward cosmology and system in philosophy is driven less by past successes than by present failures.

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RESUMEN

El segundo cuerno epistemológico del dilema de Benacerraf también puede verse como una serie de ideas metafísicas. ¿Cómo se ajustan sistemáticamente la mente, la materia y los *abstracta* absolutos (como los números) para que, por ejemplo, las mentes tengan conocimiento de los números? Ésta es una pregunta cosmológica y, puesto que parece ser inferencia de la mejor explicación por medio de la cual podamos considerar justificado el conocimiento matemático, lleva a la cuestión de la significación del sistema explicativo. ¿Esto quiere decir que, como dijera Hegel, lo real es lo racional?

[*Traducción de Francisco Hernández*]