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QUANTUM MECHANICS, CORRELATIONS, AND RELATIONAL PROBABILITY

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SUMMARY: This article sets forth and discusses the Ithaca Interpretation of Quantum Mechanics (IIQM). Section 1 presents the standard formalism of quantum mechanics and the measurement problem. Section 2 sketches Everett's interpretation as a preamble to IIQM. Section 3 sets out IIQM's central claim: it is possible to make sense of quantum mechanics by taking as the proper (and only) subject of physics the correlations among subsystems. Section 4 introduces a theorem of quantum mechanics, the SSC theorem, which supports this claim. Section 5 contends that at least two problems exist with IIQM, and one serious objection against it. Section 6 discusses a strategy based on relational probabilities to go around the objection.

KEY WORDS: measurement problem, system/subsystem, dualism, physical reality

RESUMEN: Este artículo presenta y discute la Interpretación de la Mecánica Cuántica de Ithaca (IIQM). La sección 1 expone el formalismo estándar de la mecánica cuántica y el problema de la medición. La sección 2 bosqueja la interpretación de Everett como preámbulo a la IIQM. La sección 3 plantea la tesis central de la IIQM: es posible dar sentido a la mecánica cuántica tomando como sujeto propio (y único) de la física las correlaciones entre subsistemas. La sección 4 expone el teorema SSC de la mecánica cuántica que sustenta esta tesis. En la sección 5 se sostiene que existen al menos dos problemas con la IIQM, y una seria objeción en su contra. Para sortear esta objeción, la sección 6 discute una estrategia basada en probabilidades relacionales.

PALABRAS CLAVE: problema de la medición, sistema/subsistema, dualismo, realidad física

1. *The Measurement Problem*

The traditional formulation of (non-relativistic) quantum mechanics is based on the following five principles:

(1) Representation of physical states: all possible physical states of a quantum-mechanical system S are represented by unit-length vectors in a Hilbert space.

(2) Representation of measurable properties: for each measurable property M of S, there is a linear operator $\boldsymbol{M}$ on the Hilbert space of S representing that property.

(3) Eigenvalue-eigenstate correlation: if the unitary vector in the Hilbert space representing the physical state of S is an eigenstate of the linear operator $\boldsymbol{M}$ with eigenvalue ß, then S has the value ß of property M.

(4) Dynamics: every quantum-mechanical system S evolves continuously according to the linear and deterministic Schrödinger equation, which is a function of the energy properties of the system.

(5) Collapse: if a measurement of M is made on S, then —whatever the state vector of S was prior to the measurement of M— S instantaneously and randomly *collapses* into a state in which it definitely has —in accordance with (3)— a definite value ß of M. The probability of each post-measurement state is determined by the system's initial state.

This formulation of quantum mechanics is, quite famously, *inconsistent*. According to (4), the evolution of *any* quantum-mechanical system *at all times* is governed by Schrödinger's linear and deterministic equation. But principle (5) is —manifestly— a probabilistic recipe for the violation of this equation. If one assumes that absolutely every macroscopic observer and measuring device is itself a quantum system which obeys Schrödinger's equation, as (4) demands, it follows that principles (4) and (5) predict different dynamical evolutions for *superposed* systems on measurement.

Let us consider the traditional example. Suppose that a certain observer O is measuring the x-spin of a spin-1/2 system S. And suppose that the initial state of S is a superposition of x-spin eigenstates:

$$a \,|\, x\text{-spin up} \,\rangle_S + b \,|\, x\text{-spin down} \,\rangle_S$$

According to (4), the post-measurement state of the composite system will be:

$$a \,|\text{ “spin up” }\rangle_O \,|\, x\text{-spin up} \,\rangle_S + b \,|\text{ “spin down” }\rangle_O \,|\, x\text{-spin down} \,\rangle_S.$$

But according to (5), the post-measurement state of this system will be:

$$|\text{ “spin up” }\rangle_O \,|\, x\text{-spin up} \,\rangle_S$$

with a probability of a^2, or

$$| \text{"spin down"} \rangle_O \, | x\text{-spin down} \rangle_S$$

with a probability of b^2. Now, it is clear that these are two different outcomes. The post-measurement state of $S + O$ according to (4) is still a state of *superposition*. But the post-measurement state of $S+O$ according to (5) is either a specific definite state, with a probability of a^2, where O measures "spin up", or a different definite state, with a probability of b^2, where O measures "spin down".

This is the measurement problem in quantum mechanics.

There are conflicting interpretations of the theory that purportedly solve this problem.[1] A still fashionable candidate among physicists is the von Neumann-Dirac interpretation of quantum mechanics (von Neumann 1955). In this interpretation, principle (4) in the standard formulation is replaced with this principle:

(4*) Dynamics: if no measurement of M is made on quantum-mechanical system S, then S evolves continuously according to the linear and deterministic Schrödinger equation, which is a function of the energy properties of the system.

But the catch now is that the applicability of this modified principle depends on the exact meaning of the word "measurement". When a *measurement* occurs, the wave function randomly collapses into an eigenstate of the measured property. But if no *measurement* occurs, it keeps evolving in line with the Schrödinger equation. Thus, it is clear that, unless we supplement principles (1)–(4*) and (5) with a natural and objective elucidation of this word, these five principles make quantum mechanics, if not *inconsistent* as with the original formulation, at least *incomplete*: they do not determine by themselves when principles (4*) and (5) apply. It is far from evident, however, what such a natural and objective elucidation of the word "measurement" can possibly amount to. This seems to be rather a question of how we *define* what it is to measure something. This interpretation, then, is in trouble as well. Alternative interpretations have been introduced where the applicability of (4*) and (5) rests on different, though essentially just as vague, notions such as *recording*, *macroscopic*, *information*, and so forth. Even *consciousness* has been famously put forward as a possible trigger for the random collapse of

[1] Lewis (2007) has recently defended this "orthodox" account against a "recurring heresy" according to which there is only one interpretation that solves the problem, namely, the Many-Worlds interpretation.

wave functions (Wigner 1961). None of these so-called *collapse theories*, however, offers an interpretation of quantum mechanics that makes the theory both consistent and complete. A more compelling theory in this respect is GRW (Ghirardi, Rimini, and Weber 1986). In this theory, when a measurement takes place, one component of the state vector of a system is singled out at the expense of all others (which amounts —roughly— to the collapse of the wave function, although the post-measurement system is strictly in a state of superposition) as the result of a completely natural stochastic phenomenon —which has nothing to do with our definition of what it is to *measure* something, or to *record* something, or to be *conscious*, or to be *macroscopic*, etcetera.

Another tradition in the business of solving the measurement problem is the so-called *Everett tradition*, initiated by Hugh Everett (1957a). In this tradition, there is no such thing in the world as a collapse of the wave function. Rather than trading principle (4) for something along the lines of principle (4*) in order to address the measurement problem, this tradition simply abandons principle (5) and insists that principles (1) through (4) are categorically all we need to make quantum mechanics consistent and complete. In Everett's initial enactment of the tradition, the so-called *relative-state* formulation of quantum mechanics, the usual statistical predictions of the theory —the predictions that arise from principle (5) in the old formulation— are regarded exclusively as the subjective experiences of observers who are themselves treated as ordinary physical systems. It is far from clear, however, what Everett had *precisely* in mind when he put forward this relative-state formulation. A number of related proposals —like the Many-Worlds (DeWitt 1970), Many-Minds (Albert and Loewer 1988), and Many-Histories (Gell-Mann and Hartle 1990) interpretations— have been precisely an attempt to present Everett's seminal idea in a more explicit and satisfactory way.

The third major tradition —along with Everett's proposal and the theories of collapse— in the range of alternative solutions to the measurement problem is due to David Bohm (1952). In Bohm's theory, like in Everett's tradition, it is assumed that we can dispense with principle (5) and still make full sense of quantum mechanics. Unlike Everett's proposal, however, Bohm's theory supplements principles (1)–(4) with new fundamental physical principles formulated in order to reproduce all the statistical predictions associated with (5). A distinctive feature of this theory, which John Bell has repeatedly emphasized, is that it takes wave functions to be actual physical *things*.[2] In this theory, wave functions —whose evolutions are always governed by the Schrödinger equation— are real physical objects pushing particles around, in line with these new fundamental principles (i.e. the Bohmian guidance condition), such that we always find those particles precisely where we would have expected to find them in accordance with principle (5) in the standard formulation. In Bohm's theory, then, unlike in the standard formulation, the probabilities associated with principle (5) are not a result of the intrinsic non-deterministic evolution of the physical world but rather a result of our limited epistemic access to its deterministic behavior.[3]

2. *Everett's Formulation*

The problem with Everett's formulation is, in a nutshell, that it is not at all evident how it is supposed to work. First, this formulation commits itself to some variety of modal realism: provided a physical system in a *superposed* state, all the pure states associated with it are *in some sense* independently realized. On the other hand, the subjective experiences of observers who interact with systems on measurement are always perfectly determined —although, in general, they cannot be deterministically predicted from the dynamics and the initial conditions of those systems— and do not reveal any unambiguous connection with this hypothetical "multiple realization" of pure states. Everett says:

> We shall deduce the probabilistic assertions of [principle (5)] as subjective appearances to such observers, thus placing the theory in correspondence with experience. We are then led to the novel situation in which the formal theory is objectively continuous and causal, while subjectively discontinuous and probabilistic. (1973, p. 9)

Everett maintains that it is possible to *deduce* the statistical predictions of quantum mechanics —viewed in this context as the subjective experiences of observers— from the objective formalism of the theory. This assertion is, as it stands, mysterious and insufficient. But Everett goes on to explain:

[2] The other two formulations presented here are not necessarily incompatible with this *realistic* view. But they are often associated —particularly some versions of the collapse theories— with *instrumentalist* approaches. For a different take on the Bohmian wave function, however, see Dürr *et al.* 1997.

[3] Another major tradition in the business of solving the measurement problem, which I will ignore in this paper, concerns the so-called *Modal Theories*; see, for example, Kochen 1985, Dieks 1989, and Healey 1989.

> Let one regard an observer as a subsystem of the composite system: observer + object-system. It is then an inescapable consequence that after the interaction has taken place there will not, generally, exist a single observer state. There will, however, be a superposition of the composite system states, each element of which contains a definite observer state and a definite relative object-system state. Furthermore, as we shall see, each of these relative object-system states will be, approximately, the eigenstates of the observation corresponding to the value obtained by the observer which is described by the same element of the superposition. Thus, each element of the resulting superposition describes an observer who perceived a definite and generally different result, and to whom it appears that the object-system state has been transformed into the corresponding eigenstate. In this sense the usual assertions of [principle 5] appear to hold on a subjective level to each observer described by an element of the superposition. (1973, p. 10)

Still the problem here is that there is an explanatory gap in Everett's exposition between this plurality of superposed observers, each of whom obtains a completely definite outcome on measuring some property of the object-system, and the *actual* observer detached from the quantum system under study who obtains only a particular outcome with a certain probability.[4]

As noted, there have been several attempts to recreate Everett's formulation in a more explicit and satisfactory way. In what follows, I will present and discuss a more recent addition to the growing list of such recreations: David Mermin's Ithaca Interpretation of Quantum Mechanics (IIQM) (Mermin 1998a, 1998b).

In a short paper (1957b) summarizing the results of his doctoral dissertation (1957a), where he sets forth his relative-state formulation, Everett says: "As a result of the interaction the state of the measuring apparatus is no longer capable of independent definition. It can be defined only *relative* to the state of the object-system. In other words, there exists only a correlation between the states of the two systems" (p. 457). Everett claims here, in other words, that the interaction between a measuring device and a *superposed* object-system (an interaction that leaves the composite system in a state of superposition as well), has as its most outstanding feature that neither

[4] Everett was mainly interested in a quantum theory of gravitation and cosmology, a quantum mechanics of the entire world. He believed, then, that the problem with the traditional approach is that it always treats observers as *external* to the quantum system under study, which entails that the theory is not appropriate to describe the universe as a whole, since there is nothing external to the universe (Bell 1976).

the measuring device nor the object-system can now be independently defined. There is only a *correlation* between them. This idea is at the crux of Mermin's proposal.

3. *The Ithaca Interpretation of Quantum Mechanics*

Mermin starts by arguing, in his characteristic up-front style, that he has never met an interpretation of quantum mechanics he did not dislike. And then, in order to set a framework of constraints to his own interpretation, he introduces six requirements or desiderata that any sensible interpretation of quantum mechanics should satisfy. These requirements, he admits, are motivated by his personal intuitions against the interpretations of the theory presented above. It remains to be seen, and we will get back to this later, whether his own interpretation does not violate other sensible requirements for a good interpretation. The six desiderata are the following:

(1) The theory should describe an objective reality independent of what observers know.

(2) The notion of *measurement* should play no fundamental role in the theory.

(3) The theory should be able to describe individual systems, not just ensembles.

(4) The theory should be able to describe fully isolated systems, without appealing to external perturbations.

(5) The theory should satisfy generalized Einstein-locality.

(6) The theory should rest on a (yet to be supplied) notion of objective probability.

The first requirement is clearly intended to rule out *interventionist* interpretations of quantum mechanics, such as Wigner's. *Consciousness*, then, cannot be included in the theory as a fundamental operating concept. It rules out the so-called *instrumentalist* interpretations —adopted by Heisenberg and, presumably, by Bohr— as well, according to which a wave function is nothing but a concise encapsulation of our knowledge. The second desideratum rules out the *classical* interpretations of the theory —the Copenhagen interpretation, the von Neumann-Dirac interpretation. According to it, the notion of *measurement* cannot operate in quantum mechanics as a fundamental concept either. Principles (4*) and (5) of section 1, evidently, violate this requirement. The third requirement goes essentially against the standard way of introducing the expectation values of observables in quantum mechanics —which is typically reflected in quantum mechanical textbooks.[5] According to Mermin, the theory should be able to describe individual systems because the world contains individual systems (and is one itself). The fourth desideratum rules out interpretations of the theory that rely systematically on perturbations from an external environment —as, for instance, the Many-Histories interpretation— because, again, there exists no external environment to the entire world. The fifth desideratum claims that any reasonable interpretation of quantum mechanics has to involve generalized Einstein-locality. Objectively real internal properties of an isolated individual system —as Mermin puts it— should not be altered when something is done to another non-interacting system. The sixth and final desideratum, of which we will extensively talk in sections 5 and 6, excludes any sort of hidden variables interpretation, such as Bohm's theory, from the list of plausible candidates for an acceptable interpretation. For the hidden variables interpretations, probabilities are just an epistemic feature of the theory, related to our limited access to the deterministic evolution of the world. According to Mermin, on the contrary, quantum mechanics has taught us that probabilities are more than just a convenient instrument for systematically dealing with our ignorance, but a fundamental feature of the physical world.

The main strategy of IIQM is based on the idea that quantum mechanics can be used to set a criterion of *physical reality*. Mermin says:

> Einstein used his supposition [locality], together with his intuitions about what constituted a real factual situation, to conclude that quantum mechanics offers an incomplete description of physical reality. I propose to explore the converse approach: assume that quantum mechanics does provide a complete description of physical reality, insist on generalized Einstein-locality, and see how this constraints what can be considered physically real. (1998a, p. 552)

Thus, the strategy of IIQM is to take the formalism of quantum mechanics as given, and deduce from the theory itself what quan-

[5] Griffiths, for instance, asserts: "In short, the expectation value is the average of repeated measurements on an ensemble of identically prepared systems, not the average of repeated measurements on one and the same system" (1995, p. 15).

tum mechanics is trying to tell us about physical reality —and not the converse, namely, to adopt a general view of the world and try to insert quantum mechanics into it. More precisely, we must adopt the formalism consisting of principles (1)–(4) of section 1, without any principle of collapse, and enforce upon it the set of requirements (1)–(6) of this section. By doing so, we will unravel what quantum mechanics is trying to say about physical reality.

There is a crucial distinction in IIQM, a distinction that has been implicitly used here, between the notions of *reality* and *physical reality*. As just noted, IIQM seeks to get from the quantum-mechanical formalism a criterion of physical reality, but not a criterion of (unqualified) reality. According to Mermin, physical reality is *narrower* than what is real to the conscious mind. Quantum mechanics is certainly about physical reality, but it is not about (unqualified) reality. To put it differently, quantum mechanics is not a fundamental theory of *everything* (we leave aside here, of course, *more fundamental* quantum theories like quantum field theory, quantum string theory, supergravity, and so on) if *everything* is to include consciousness.

Mermin offers the following example to make this point clear. Suppose that observer O is looking at some blue object. And suppose, additionally, that O is not color blind and has, then, a *sensation* when she looks at the object. For Mermin, O's sensation of blueness, or *qualia*, is real, but not physically real. Physics can indeed talk about certain classes of spectral densities of the radiation field. It can speak of the stimulation of a number of receptors within the eye. It can describe how nerve impulses go from the eye to the visual cortex. But it is completely and absolutely silent about the *qualia* of blueness. The point here, very succinctly, is that quantum mechanics is not about what is real *to us*, but about what is *physically* real.

The strategy that IIQM follows to solve the measurement problem, then, is to take the quantum-mechanical formalism as given and determine what that, along with some constraints on the possible results, has to say about *physical reality* —and not, once again, about (unqualified) *reality*, which is beyond the scope of the theory and physics in general.

Now, the question here is: where does one look in the quantum-mechanical formalism to hear this pronouncement, as it were, on physical reality?

4. *The SSC Theorem*

Mermin presents a theorem of quantum mechanics, a theorem he deems extremely important and not enough studied, about the relation between the state of a system and its corresponding subsystems. It is the theorem of the Sufficiency of Subsystem Correlations (SSC theorem). It says the following:

> SSC theorem: subsystem correlations, for any resolution of a particular system into subsystems, are enough to determine the state of the entire system uniquely.

By *systems* and *subsystems*, Mermin simply refers to the traditional representation of a complex system by products of subsystem state spaces. If the system is (say) a Heisenberg model of certain number of magnetic ions, the subsystems are the spin degrees of freedom of those individual ions. If the system is a hydrogen atom, then the subsystems are the electron and the proton, further resolved, if this is of interest, into their spin and orbital degrees of freedom. The notion of *correlation* is defined in the following way:

> Correlations: the correlations among subsystems are the mean values, at a time, of all the system's observables that consist of products over subsystems of individual subsystem observables.

Then, what the SSC theorem states is that it is sufficient to have the mean values of all these product-over-subsystem observables in order to compute the mean values of whatever set of global system observables is needed to pin down the state of the whole system. The proof of the theorem is straightforward. It immediately follows from these three premises:

(i) The means of all observables for the entire system determine its state.

(ii) The set of all products over subsystems of subsystem observables contains a basis for the algebra of all such system-wide observables.

(iii) The algorithm that supplies observables with their mean values is linear on the algebra of observables.

Then, based on these premises, Mermin claims that the quantum state of a complex system *is nothing more than* a brief encapsulation of all the correlations among its subsystems. In other words, anything you can say in terms of the quantum state of a complex system can be entirely and accurately translated into statements describing correlations among its subsystems. And then, following the general IIQM strategy stated above, Mermin concludes the only proper subject for the physics of a system is the correlations among its subsystems. That is, the physical reality of any system is fully contained in (i) the correlations among its subsystems (internal correlations), and (ii) the correlations with other systems, viewed all together as subsystems of a larger system (external correlations). According to IIQM, hence, correlations have physical reality, that which they correlate does not. This conclusion, of course, does not entail that these correlata do not have (unqualified) reality, or *conscious* reality —observers, after all, are always confident in their obtaining definite outcomes on a measurement. But it does entail that these correlata are not physically real.

Now, how does IIQM allegedly solve the measurement problem in quantum mechanics? First, IIQM observes (correctly) that a measurement is just a particular kind of correlation between two particular kinds of subsystems: a *specimen* and an *apparatus*. But, as we have just learned, for IIQM physics is just about correlations and not about correlata. Therefore, IIQM claims —plainly and simply— that there is no such thing as the measurement problem in quantum mechanics: the Schrödinger equation is the whole story and there is no need for any principle of collapse. This is not to say that there is no problem whatsoever. The ability of consciousness to go beyond its own correlations to a direct perception of its own underlying correlata is, for Mermin, a deep puzzle. But this is not a problem for quantum mechanics. It is a problem that has to do with the mysteries of conscious awareness, and its solution, if any, is beyond quantum mechanics and physics.

Let us get back for a minute, to look at this more closely, to the system S of section 1. An observer O is measuring the x-spin of S. And the initial state of S is a superposition of x-spin eigenstates. According to IIQM, and provided the linearity of the dynamical equations of motion, the post-measurement state of the composite system $O + S$ will be:

$$a\,|\text{ “spin up” }\rangle_O\,|\,x\text{-spin up }\rangle_S + b\,|\text{ “spin down” }\rangle_O\,|\,x\text{-spin down }\rangle_S.$$

The measurement problem arises when trying to reconcile this kind of result with the incontrovertible fact that observers always believe to have definite outcomes on their measurements. In our case, after a measurement of the x-spin of S takes place, observer O definitely believes that either "spin up" or "spin down" is the case. But IIQM tries to *dissolve* (if not *resolve*) the measurement problem simply by denying that there are absolute matters of fact about O measuring "spin up" or O measuring "spin down", and also denying that there are absolute matters of fact about S being spin up or S being spin down. For IIQM, O measures "spin up" *relative to S* being spin up and O measures "spin down" *relative to S* being spin down. And, given the symmetric nature of this correlation, S is spin up *relative to O* measuring "spin up" and S is spin down *relative to O* measuring "spin down". Our profound conviction as observers that we obtain definite outcomes on all measurements is, in IIQM, just a result of our being *conscious* observers. It is our *consciousness* what enables us to move beyond the physical correlations we are part of to a direct perception of the underlying correlata. But physics, for IIQM, is just about correlations. Accordingly, this mysterious ability to perceive correlata, though a deep puzzle, is not a problem for quantum mechanics.

5. *Two Problems and One Objection*

There are at least two problems (or, at best, perplexities) with IIQM. These problems are not, of necessity, fatal. Perhaps, in order to accept IIQM, we will simply have to bite the bullet and get used to them, as we have got used in the past to extremely perplexing things revealed by fundamental physics.

The problems are the following:

(a) Mental-physical dualism: IIQM explicitly relies on a dualist conception of the world. It certainly respects, however, the causal closure of the physical world, so this dualism is a *moderate dualism*.[6] But it entails mind-body dualism nonetheless. Brains can be fully described (at least in principle) quantum-mechanically. Minds are beyond the scope of quantum mechanics.

(b) Absence of correlata: in IIQM, correlations and only correlations have physical reality. Individual subsystems (correlata) do not have physical reality.

[6] Unlike (say) a Cartesian or Wignerian interventionist dualism, which disrupts the causal closure of the physical world.

As noted, I think these problems are not fatal. There are indeed some philosophers who welcome mind-body dualism as a corollary of quantum mechanics.[7] However, it seems to me, regarding (a), that scientific theories should be as less involved with philosophical commitments as possible. Or, maybe more realistically, given that most theories carry their ontological and metaphysical commitments, they ought to be as less involved with *controversial* philosophical commitments as possible. But again, we must acknowledge that the special and general theories of relativity have taught us novel and perplexing things about space and time. Perhaps quantum mechanics is trying to teach us new things about our minds as well.

Things get worse, however, in terms of (b). This philosophical commitment is still, I think, more controversial than mind-body dualism. One of our original and most vigorous intuitions concerning the world is that there are *things* out there, and we are *things* ourselves, and that the physical reality of all these *things* (if we leave aside all solipsistic suspicions) is an incontrovertible, autonomous, independent fact of the world. According to (b), however, this basic intuition is false. All the tables and chairs out there are not physically real, but merely an outgrowth of our minds. Their internal and external correlations, indeed, are physically real. But the physical reality of the tables and chairs themselves is nothing more than just a persistent and shared delusion. As just noted, I take this to be a highly contentious philosophical commitment. But, once again, maybe quantum mechanics is just trying to teach us how mistaken we have been regarding some of our key intuitions about the world.[8]

IIQM faces a major objection, nevertheless, because of the way in which it deals with *probabilities*. Mermin argues that, in the context of IIQM, it is possible to make sense of quantum mechanics conditional on eventually making sense of the notion of objective probability —as stipulated by requirement (6) of section 3. But it is not clear the role that objective probability might play in IIQM. And it manifestly begs the question to argue, as Mermin does, that this role would be more evident if we had a better understanding of objective probability.

The so-called *problem of probability* haunts most no-collapse interpretations of quantum mechanics in Everett's tradition. In Many-

[7] David Chalmers (1996), for instance, has famously displayed such a welcoming attitude toward the (alleged) dualist consequences of quantum mechanics.

[8] Mermin appears to acknowledge the significance of this problem in Mermin 1999, where he attempts —without much success— to address it.

Worlds theory, for example, the problem assumes the following form. Given our system S, whose initial state is a superposition of x-spin eigenstates:

$$a \,|\, x\text{-spin up } \rangle_S + b \,|\, x\text{-spin down } \rangle_S.$$

If a measurement of x-spin is carried out on S by O, the post-measurement state of the composite system $O + S$ will correspond to two distinct worlds, one in which O measures "spin up" and one in which O measures "spin down":

$$\text{[world 1]} \qquad |\text{ "spin up" } \rangle_O \,|\, x\text{-spin up } \rangle_S$$

$$\text{[world 2]} \qquad |\text{ "spin down" } \rangle_O \,|\, x\text{-spin down } \rangle_S$$

Quantum mechanics predicts that a measurement of x-spin will come out "spin up" with probability a^2 and "spin down" with probability b^2. But the question is: does it make any sense to cast this prediction provided that there is nothing that O ignores, prior to the measurement, about its outcome? O will certainly measure "spin up" in world 1 and "spin down" in world 2. It is part and parcel of the Many-Worlds theory that neither of these worlds is the *real*, or *original*, or *true* world. And it is part and parcel of the theory that neither of these Os is the *real*, or *original*, or *true* observer. These are clearly two worlds in which two observers obtain distinct outcomes from an x-spin measurement. It makes no sense, consequently, to either maintain that the probability of O ending up in a state of believing "spin up" is Φ (whatever Φ is), or instead that the probability of O ending up in a state of believing "spin down" is Ω (whatever Ω is).

Exactly the same kind of problem arises in IIQM. What does it mean to say that the probability of O measuring "spin up" relative to S being spin up is Φ, while the probability of O measuring "spin down" relative to S being spin down is Ω? As noted in relation to the Many-Worlds theory, there is nothing at all which O ignores concerning the outcome of this measurement. Both correlations will, as a matter of physical fact, be realized. O may ignore, perhaps, whether *she* (her own true self, her diachronic continuous *mind* —assuming there is such a thing) will end up believing "spin up" or "spin down". But this, according to IIQM, cannot be of any interest to quantum mechanics, since it is our being *conscious* —which is beyond the scope of quantum mechanics— what makes us believe that we have a definite outcome: either "spin up" or "spin down". Moreover, as established by requirement (6) of section 3, the notion of probability that operates in IIQM must be that of *objective* probability. For this reason, the statistical predictions of quantum mechanics cannot be in this interpretation a consequence of our peculiar epistemic powers or limitations.

This constitutes, then, a serious objection to IIQM. It is not clear which role the standard statistical predictions of quantum mechanics play in the theory. And it begs the question, as argued before, to claim that this role would be more evident if we had a better grasp of objective probability, since —as established by requirement (6)— IIQM is a theory that aspires to make sense of quantum mechanics conditional on eventually making sense of the notion of objective probability.

But what if an adequate notion of objective probability were available? For Mermin, again, this would guarantee the adequacy of IIQM. We can take a further step, then, and give IIQM one more shot by assuming that there exists such a notion. So let us take the best candidate in the market, as far as IIQM is concerned, and see whether it really gives support to Mermin's proposal. I believe a suitable candidate to carry out this test is Simon Saunders's notion of *relational* probability.

6. *Relational Probability*

Saunders (1998) claims that a relational account of probability can be used to solve the *problem of probability* in all interpretations of quantum mechanics inspired by Everett's ideas. He says:

> On the contrary, I claim that the problem of probability can be fully resolved in Everett's framework. What is needed is a thorough-going relativization of physical modal attributes, specifically of value-definiteness and probability, viewed as an extension of the relativization of tense familiar to classical physics. (1998, pp. 374–375)

The standard concept of probability, affirms Saunders, applies only to a situation in which one specific possibility, a_i, out of a range of alternative possibilities $a_1, a_2, \ldots, a_n$ is true, or is realized, or actually occurs, such that it excludes all others. And this is precisely what the *Everettish* theories deny. And this is exactly why the problem of probability arises for them in the first place.

Let us stop for a minute to look at this more closely. It is generally assumed that a probability statement like:

a_i has probability p

simply means that, given the necessary conditions:

> a_i will happen with probability p.

But given the standard notion of probability, this implies that:

> if a_i happens, then $a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n$ will not happen.

But, for those theories inspired by Everett's ideas, in some sense or other:

> a_i happens, and $a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n$ also happen.

Thus, in this classical understanding of probability, Everett's approach to quantum mechanics, as well as the theories inspired by it, is inconsistent.[9]

Now, according to Saunders, the traditional ideas of *identity* and *substance* play a tacit but decisive role in this standard way of understanding probability. But, he says, the theory of relativity has shown us that these concepts have no reference. There is no such thing as a *substance*, the substratum of changing attributes which does not itself change. And, without such a concept, the notion of identity over time (as something different from *gen-identity* or similar notions derived from criteria of physical spatio-temporal continuity) goes by the board.

What does this have to do with probability? For Saunders, this shows that it is time to replace the old notion of probability with a new one, a *relational* one, in which the context of use, the context in which a statement like "a_i has probability p" is uttered, is made explicit.

For this relational account, then, a probability statement like:

> a_i has probability p

really means something like:

> a_i has probability p relative to z.

And this, in turn, means that, given the necessary conditions:

[9] There are some ways in which, even preserving the standard notion of probability, Everett's tradition manages to avoid inconsistency, though committing itself to some new problems and perplexities. Many-Minds theory is precisely an example of that.

> a_i comes after z and will happen with probability p relative to z.

Given the relational account of probability, this implies that:

> if a_i happens relative to z, then $a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n$ will not happen relative to z.

But distinct events must not, of necessity, be exclusively relativized to one singular event. And this, therefore, does not rule out the possibility of:

> a_i happens relative to z_i, and a_1 happens relative to z_1, a_2 happens relative to $z_2, \ldots, a_{i-1}$ happens relative to z_{i-1}, a_{i+1} happens relative to $z_{i+1}, \ldots, a_n$ happens relative to z_n,

where $z_1, z_2, \ldots, z_n$ are different events to which events $a_1, a_2, \ldots, a_n$ are respectively relativized. And so, this new relational account of probability, unlike the traditional account, allows for a consistent *Everettish* approach to quantum mechanics. And, in particular, this relational account of probability allows (allegedly) for a consistent IIQM approach to the theory.

What should we make of this argument?

Saunders objects to Albert and Loewer when they maintain that the cost of surrendering the "trans-temporal identity of minds" (1988, p. 211) is that we can no longer make sense of statements like "the probability that I will obtain 'spin up' on measurement of S is p". Saunders, in contrast, holds that it is possible to make sense of such statements in the *Everettish* approach precisely by rebutting any *substantial* notion of identity over time. Accordingly, he translates the previous statement into: "the probability relative to I at t_1 that I at t_2 observe 'spin up' on measurement of S is p". Assuming there is no "trans-temporal identity of minds", I at t_1 is not identical to I at t_2. But then, following the previous reasoning, we can establish that different events, like S being spin up or spin down, must not necessarily be relativized to one singular event, like I at t_2 observing "spin up" or I at t_2 observing "spin down". We can conclude, therefore, that there is no inconsistency in maintaining that S is spin up relative to I at t_2 measuring "spin up" while claiming that S is spin down relative to I at t_2 measuring "spin down", exactly like IIQM predicted.

The problem here, in my view, is that Saunders is wrong in maintaining that this preserves the idea that probabilities have an objective meaning. In other words, I believe, along with Albert and Loewer, that this relational account of probabilities turns statements like "the probability that I will obtain X on measurement of Q is p" into completely vacuous statements. What, after all, are these probabilities exactly predicting? There is nothing that "I" does not know concerning the outcome of this measurement. So nothing about it can be, even from a purely objective perspective, *probable*, but *certain*.

Saunders, however, observes:

> Many philosophers take the peculiarities of the various relational readings of these sentences as evidence for the failing of relationalism; but equally, we could conclude that our ordinary conception of change is muddled, and involves much else besides physics. How are we to picture the process of probabilistic becoming? I say that it is to be understood as a system of relations, the same here as with deterministic becoming, in which notions of space-time and probability function as primitives. The "problem of probability", so-called, is the problem of how to provide something more. But we have learned to live with this lacuna, in the deterministic case, and we can do the same in quantum mechanics. (1998, pp. 378–379)

I find this ineffective. First, this takes us right back to where we started. We could not find a satisfactory account of objective probability for IIQM. And this requires us to accept the relational account and believe that, somehow, it works. Second, the plausibility of IIQM depends, as Mermin pointed out, precisely on our discovering a reasonable account of objective probability. And this stipulates that we should not try to get deeper into this issue, but take instead the relational notion as a primitive and learn to live with this "lacuna".

Let us get back now, setting aside the shortcomings of Saunders's proposal, to our original strategy.

Let us assume for a minute that this relational account does indeed solve the problem of probability in IIQM. We wondered before whether, given a reasonable account of objective probability, IIQM would become an adequate interpretation of quantum mechanics. That is exactly what, as noted before, Mermin suggests. So let us assume for a moment, again, that we are in possession of such plausible account. Is IIQM, then, an adequate interpretation of the theory? In other words: does IIQM satisfy, not only Mermin's six desiderata presented before, but also the fundamental requirement of *plausibility*? The answer, I think, is far from conclusive, and so hardly what Mermin anticipated.

I argued in the preceding section that two problems, mind-body dualism and the absence of correlata, make IIQM, if not unacceptable, at least controversial. But now, while assuming that we are in possession of a reasonable account of objective probability, we find ourselves faced with a third contentious corollary of IIQM: the abandonment of the "trans-temporal identity of minds". Leaving aside the question of whether this is compatible with IIQM's explicit dualism, which in itself does not seem to be plausible,[10] I believe that these three corollaries put together make IIQM extremely unlikely. As just noted, and under the (false) assumption that Saunders's relational account of probability solves the problem of probability for IIQM, I think IIQM cannot be finally, completely, and unequivocally discarded. When compared, though, with other competing interpretations of quantum mechanics, like GRW and Bohm's theory, where no such disruptive corollaries appear to follow, it clearly and unambiguously looks extravagant and weak.

REFERENCES

Albert, D. and B. Loewer, 1988, "Interpreting the Many-Worlds Interpretation", *Synthese*, vol. 77, pp. 195–213.

Bell, J., 1976, "The Measurement Theory of Everett and de Broglie's Pilot Wave", in J. Bell, 1987, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, Cambridge, pp. 93–99.

Bohm, D., 1952, "A Suggested Interpretation of Quantum Theory in Terms of Hidden Variables", *Physical Review*, vol. 85, pp. 166–193.

Chalmers, D., 1996, *The Conscious Mind*, Oxford University Press, Oxford.

DeWitt, B., 1970, "Quantum Mechanics and Reality", *Physics Today*, vol. 23, pp. 30–35.

Dieks, D., 1989, "Resolution of the Measurement Problem Through Decoherence of the Quantum State", *Physics Letters A*, vol. 142, pp. 439–446.

Dürr, D., S. Goldstein and N. Zanghi, 1997, "Bohmian Mechanics and the Meaning of the Wave Function", in R. Cohen, M. Hornen and J. Stachel (eds.), *Experimental Metaphysics: Quantum Mechanical Studies for Abner Shimony, Volume I*, Kluwer, Dordrecht, pp. 25–38.

Everett, H., 1973, "The Theory of the Universal Wave Function", in B. DeWitt and N. Graham (eds.), *The Many-Worlds Interpretation of Quantum Mechanics*, Princeton University Press, Princeton, pp. 3–140.

———, 1957a, *On the Foundations of Quantum Mechanics*, PhD Dissertation, Princeton University.

[10] This would amount, in any case, to the contention that our minds are, as it were, disconnected and ephemeral outbursts of mental *stuff* —whatever that is— that constantly and pervasively transpire from the physical world.

Everett, H., 1957b, "Relative State Formulation of Quantum Mechanics", *Reviews of Modern Physics*, vol. 29, pp. 454–462.

Gell-Mann, M. and J. Hartle, 1990, "Quantum Mechanics in the Light of Quantum Cosmology", in W. Zurek (ed.), *Complexity, Entropy, and the Physics of Information*, Addison-Wesley, Reading, pp. 425–459.

Ghirardi, G., A. Rimini, and T. Weber, 1986, "Unified Dynamics for Microscopic and Macroscopic Systems", *Physical Review D*, vol. 34, pp. 470–491.

Griffiths, D., 1995, *Introduction to Quantum Mechanics*, Prentice Hall, New York.

Healey, R., 1989, *The Philosophy of Quantum Mechanics*, Cambridge University Press, Cambridge.

Kochen, S., 1985, "A New Interpretation of Quantum Mechanics", in P. Mittelstaedt and P. Lahti (eds.), *Symposium on the Foundations of Modern Physics*, World Scientific, Singapore, pp. 151–169.

Lewis, P., 2007, "Empty Waves in Bohmian Quantum Mechanics", *British Journal for the Philosophy of Science*, vol. 58, pp. 787–803.

Mermin, D., 1999, "What Do These Correlations Know about Reality?", *Foundations of Physics*, vol. 29, pp. 571–587.

———, 1998a, "The Ithaca Interpretation of Quantum Mechanics", *Pramana*, vol. 51, pp. 549–565.

———, 1998b, "What is Quantum Mechanics Trying to Tell Us?", *American Journal of Physics*, vol. 66, pp. 753–767.

Neumann, J. von, 1955, *Mathematical Foundations of Quantum Mechanics*, Princeton University Press, Princeton.

Saunders, S., 1998, "Time, Quantum Mechanics, and Probability", *Synthese*, vol. 114, pp. 373–404.

Wigner, E., 1961, "Remarks on the Mind-Body Question", in E. Wigner, 1967, *Simmetries and Reflections*, Indiana University Press, Bloomington, pp. 171–184.

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